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ATTITUDE CONTROL FOR A SMALL SPACECRAFT USING MAGNETORQUERS



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Classification of Spacecrafts

Class	Mass, kg
Large spacecraft	>1000
Small spacecraft	<500
Minispacecraft	100-500
Microspacecraft	10-100
Nanospacecraft	1-10
Picospacecraft	0,1-1

Control of spacecraft motion relative to the center of mass

Determination of the angular velocity and the angular position of the spacecraft

Stabilization of the spacecraft angular velocity

The orientation of the spacecraft to the desired angular position

Disturbance torque	PRISM (8,5 кг)	ASUKA (420 кг)
Gravity-Gradient torque	$8 \cdot 10^{-7}$ N*m	$1 \cdot 10^{-3}$ N*m
Aerodynamic Drag	$3 \cdot 10^{-8}$ N*m	$1 \cdot 10^{-4}$ N*m
Solar Pressure torque	$1 \cdot 10^{-8}$ N*m	$4 \cdot 10^{-5}$ N*m
Residual Magnetic torque	$3 \cdot 10^{-6}$ N*m	$5 \cdot 10^{-5}$ N*m

Residual Magnetic moment

Estimation of the residual magnetic moment of the spacecraft Nano-JASMINE (T. Inamori et al. 2011)

Type of residual magnetic moment	Source	Magnitude (A*m^2)
A permanent magnetic moment	Magnetic materials and electrical circuits	$1*10^{-1}$
The magnetic moment depending on the orbital period	Heater Solar panels	$1*10^{-3}$ $1*10^{-4}$ $6*10^3$
Magnetic moment depending on the frequency of control	Electronic circuit (a device that converts the input analog signal to a digital signal) Electronic circuit (flywheels) Electronic circuit (power supply system)	$1*10^6$ $1*10^5$ $3*10^3$
Other	Magnetotorquers Flywheels Transceivers	$2*10^6$ $1*10^4$ $1*10^3$

Magnetic attitude system

The main characteristics of the magnetic orientation system

Advantages	Disadvantages
Reliability <ul style="list-style-type: none">• No moving parts• Low degree of degradation	The difficulty of achieving a three-axis orientation of the spacecraft
Low cost	Time-variable geomagnetic field
Low power consumption	

Tasks of research

1. Evaluation of the influence of the residual magnetic moment on the algorithm for stabilizing the angular velocity of the small spacecraft;
2. Development of an attitude control algorithm for the small spacecraft on the base of linear PD controller to compensate the residual magnetic moment;
3. Development of a control algorithm for the small spacecraft on the base of sliding mode.

MATHEMATICAL MODEL OF THE SMALL SPACECRAFT MOTION

Dynamics and kinematics

$$\omega_{bi_x}^b = \frac{1}{I_x} [(I_y - I_z) \omega_{bi_y}^b \omega_{bi_z}^b + M_{grav_x}^b + M_{a_x}^b + M_{res_x}^b]$$

$$\omega_{bi_y}^b = \frac{1}{I_y} [(I_z - I_x) \omega_{bi_x}^b \omega_{bi_z}^b + M_{grav_y}^b + M_{a_y}^b + M_{res_y}^b]$$

$$\omega_{bi_z}^b = \frac{1}{I_z} [(I_x - I_y) \omega_{bi_x}^b \omega_{bi_y}^b + M_{grav_z}^b + M_{a_z}^b + M_{res_z}^b]$$

$$\vec{\omega}_{bi}^b = \vec{\omega}_{bo}^b + \vec{\omega}_{oi}^b = \vec{\omega}_{bo}^b + R_b^o \vec{\omega}_{oi}^o$$

$$R_o^b = \begin{bmatrix} q_0^2 - q_1^2 - q_2^2 + q_3^2 & 2(q_0q_1 - q_2q_3) & 2(q_0q_2 - q_1q_3) \\ 2(q_0q_1 - q_2q_3) & -q_0^2 + q_1^2 - q_2^2 + q_3^2 & 2(q_1q_2 - q_0q_3) \\ 2(q_0q_2 - q_1q_3) & 2(q_1q_2 - q_0q_3) & -q_0^2 - q_1^2 + q_2^2 + q_3^2 \end{bmatrix}$$

$I = \{I_x, I_y, I_z\}$ - diagonal (3x3) matrix of small satellite inertia tensor

- $\vec{\omega}_{bo}^b$ - angular velocity of small satellite in the body coordinate system
- $\vec{\omega}_{oi}^o$ - angular velocity of the orbital coordinate system relative to an inertial coordinate system
- $\vec{\omega}_{bo}^b$ - angular velocity of the the body coordinate system relative to the orbital coordinate system

$$\dot{q}_{0_{bo}} = \frac{1}{2} (-\omega_{bo_x}^b q_{1_{bo}} - \omega_{bo_y}^b q_{2_{bo}} - \omega_{bo_z}^b q_{3_{bo}})$$

$$\dot{q}_{1_{bo}} = \frac{1}{2} (-\omega_{bo_x}^b q_{0_{bo}} + \omega_{bo_z}^b q_{2_{bo}} - \omega_{bo_y}^b q_{3_{bo}})$$

$$\dot{q}_{2_{bo}} = \frac{1}{2} (\omega_{bo_y}^b q_{0_{bo}} + \omega_{bo_x}^b q_{3_{bo}} - \omega_{bo_z}^b q_{1_{bo}})$$

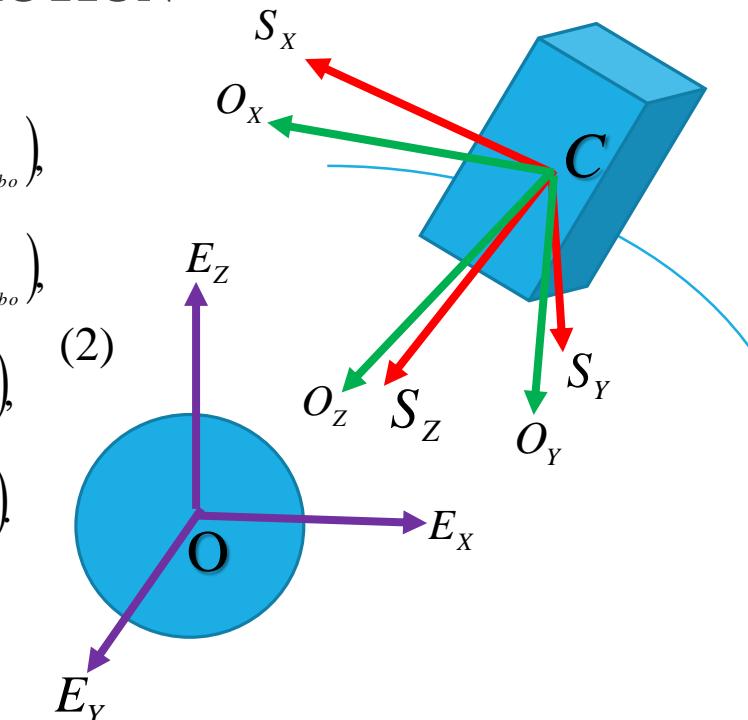
$$\dot{q}_{3_{bo}} = \frac{1}{2} (\omega_{bo_z}^b q_{0_{bo}} + \omega_{bo_y}^b q_{1_{bo}} - \omega_{bo_x}^b q_{2_{bo}})$$

$$\vec{q}_{bo} = [q_{0_{bo}}, q_{1_{bo}}, q_{2_{bo}}, q_{3_{bo}}]$$

$$M_{grav_x}, M_{grav_y}, M_{grav_z}$$

$$M_{a_x}, M_{a_y}, M_{a_z}$$

$$M_{res_x}, M_{res_y}, M_{res_z}$$



- quaternion, setting the current angular position of the small satellite in the orbital coordinate system

- Gravity-Gradient torque

- Magnetic control torque

- Residual Magnetic torque

External disturbances

Control torque

Gravity-Gradient torque

$$\overline{M}_{grav} = 3\omega_0^2 \begin{bmatrix} 2(I_z - I_y)(q_2q_3 + q_0q_1)(1 - 2(q_1^2 + q_2^2)) \\ 2(I_x - I_z)(1 - 2(q_1^2 + q_2^2))(q_1q_3 + q_0q_2) \\ 4(I_y - I_x)(q_1q_3 + q_0q_2)(q_2q_3 + q_0q_1) \end{bmatrix} \quad (3)$$

Residual Magnetic torque

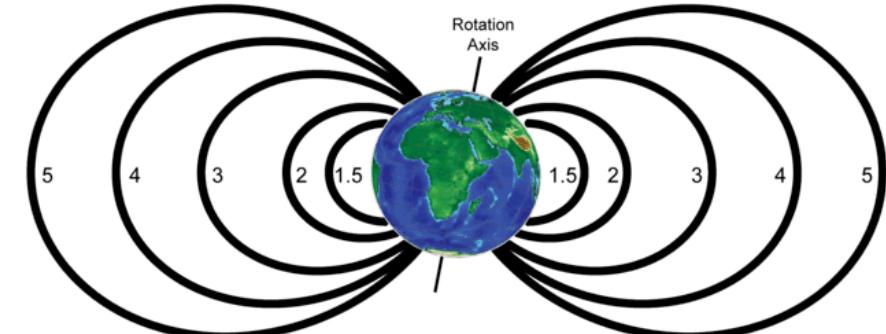
$$\vec{m}_{res} = [m_{res_x}, m_{res_y}, m_{res_z}] \quad (4)$$

$$\overrightarrow{\mathbf{M}}_{res} = \vec{m}_{res} \times \vec{B} \quad (5)$$

Magnetic torque

$$\overrightarrow{m_a} = [m_{a_x}, m_{a_y}, m_{a_z}] \quad (6)$$

$$\overrightarrow{\mathbf{M}}_a = \vec{m}_a \times \vec{B} \quad (7)$$



Earth's magnetic field (IGRF model)

$$\vec{B} = \mu_0 \vec{H} \quad (8)$$

$$\vec{H} = -\nabla V \quad (9)$$

$$V(r, \theta, \varphi) = R \sum_{n=1}^k \left(\frac{R}{r} \right)^{n+2} \sum_{m=0}^n [g_n{}^m \cos(m\varphi) + h_n{}^m \sin(m\varphi)] P_n{}^m(\theta) \quad (10)$$

$$H_r = -\frac{\partial V}{\partial r} = \sum_{n=1}^k \left(\frac{R}{r} \right)^{n+2} (n+1) \sum_{m=0}^n [g_n{}^m \cos(m\varphi) + h_n{}^m \sin(m\varphi)] P_n{}^m(\theta),$$

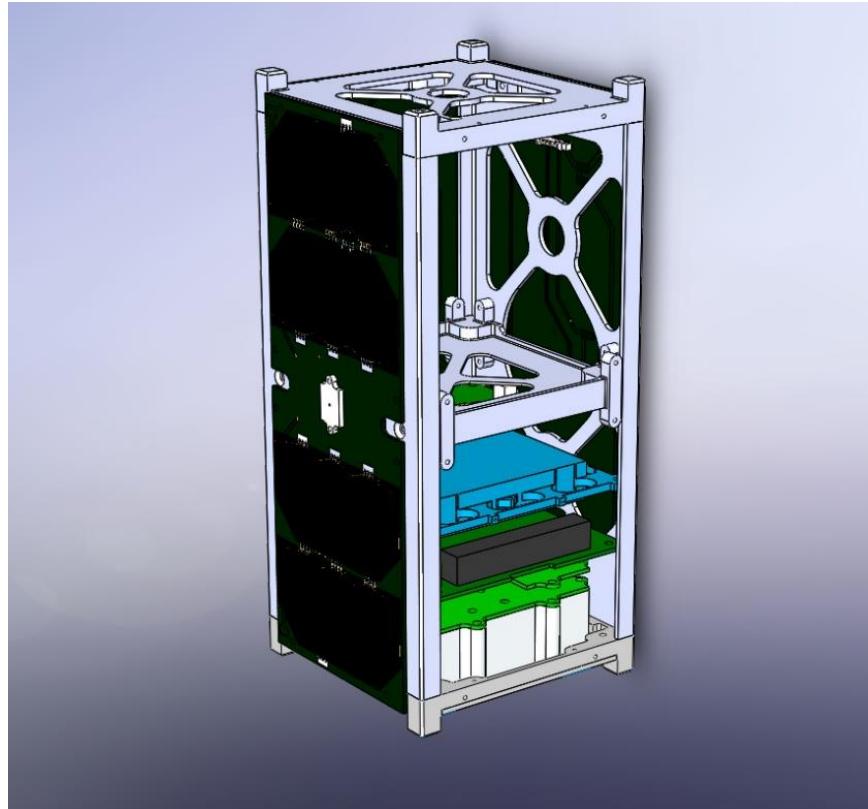
$$H_\theta = -\frac{1}{r} \frac{\partial V}{\partial \theta} = -\sum_{n=1}^k \left(\frac{R}{r} \right)^{n+2} \sum_{m=0}^n [g_n{}^m \cos(m\varphi) + h_n{}^m \sin(m\varphi)] \frac{\partial P_n{}^m(\theta)}{\partial \theta}, \quad (11)$$

$$H_\varphi = -\frac{1}{r \sin \theta} \frac{\partial V}{\partial \varphi} = -\frac{1}{\sin \theta} \sum_{n=1}^k \left(\frac{R}{r} \right)^{n+2} \sum_{m=0}^n [-m g_n{}^m \sin(m\varphi) + m h_n{}^m \cos(m\varphi)] P_n{}^m(\theta),$$

$$\overrightarrow{B}^o = R_b^o \overrightarrow{B}^b \quad (12)$$

r, θ, φ - are geocentric coordinates
 $R = 6371.2 \cdot 10^3 m$ - is a reference radius for the Earth

$g_n{}^m$ $h_n{}^m$ - Earth Magnetic Field Model IGRF coefficients
 $\mu_0 = 7.9 \times 10^{15} Wb \cdot m$



$$x = 2804.7 \text{ km}$$

$$y = 5065.2 \text{ km}$$

$$z = 4157.7 \text{ km}$$

$$v_x = 3.23 \text{ km/s}$$

$$v_y = 3.07 \text{ km/s}$$

$$v_z = -5.99 \text{ km/s}$$

$$I_x = 0.0505 \text{ kg} \cdot \text{m}^2$$

$$I_y = 0.0505 \text{ kg} \cdot \text{m}^2$$

$$I_z = 0.0109 \text{ kg} \cdot \text{m}^2$$

The small spacecraft orbit is 600 km, the period is 6024 s

Spacecraft Stabilization Algorithm - B-dot

$$\vec{m} = -K_d \dot{\vec{B}}_i \quad (13)$$

$$\dot{\vec{B}}_{i,k} \approx \frac{\vec{B}_{i,k} - \vec{B}_{i,k-1}}{\Delta t} \quad (14)$$

K_d - control gain, positive constant

Spacecraft Stabilization Algorithm – «B-dot bang-bang»

$$\vec{m} = -m_{\max} \begin{pmatrix} \text{sign}(\dot{B}_x) \\ \text{sign}(\dot{B}_y) \\ \text{sign}(\dot{B}_z) \end{pmatrix} \quad (15)$$

$$\vec{m} = \begin{cases} m_{\max} \ npu & \dot{\vec{B}}_i < 0, \\ -m_{\max} \ npu & \dot{\vec{B}}_i > 0. \end{cases}$$

B_i - magnetic field vector component

Spacecraft Stabilization Algorithm – «Follow B-field»

$$\vec{m} = m_{\max} \begin{pmatrix} -\text{sign}(\dot{B}_x) \\ -\text{sign}(\dot{B}_y) \\ \frac{1}{2} \end{pmatrix} \quad (16)$$

m_{\max} - maximum value of the magnetic moment of the magnetotorquer

Control algorithms	Angular velocity damping time, s	Stabilization accuracy (angular velocity module, rad/s) ($w_0=0,4$ rad/s)
Spacecraft Stabilization Algorithm - «B-dot»	9936	0,1891
Spacecraft Stabilization Algorithm - «B-dot bang-bang»	1867	0,1891
Spacecraft Stabilization Algorithm - «Follow B-field»	2450	0,1891

a) $\bar{m}_{res} = [0; 0; 0]$ b) $\bar{m}_{res} = [0,005;0; 0]$ c) $\bar{m}_{res} = [0,05; 0; 0]$ d) $\bar{m}_{res} = [0,5; 0; 0] \quad (A \cdot m^2)$

(T. Inamori et al. 2011)

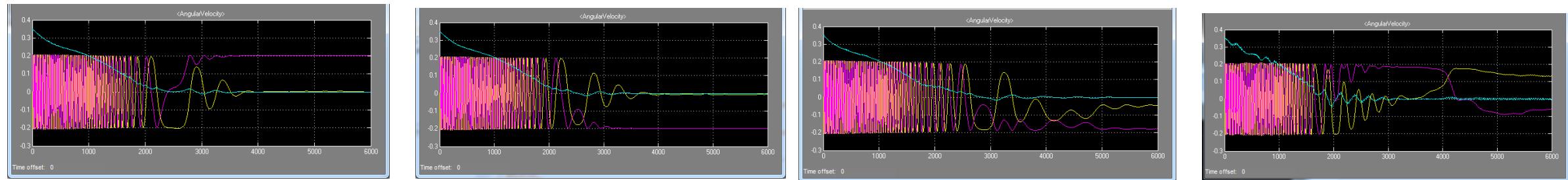


Figure 1 – Change of angular velocity of the spacecraft by $I = [0,0505; 0,0505; 0,0109] \text{ kg} \cdot \text{m}^2$

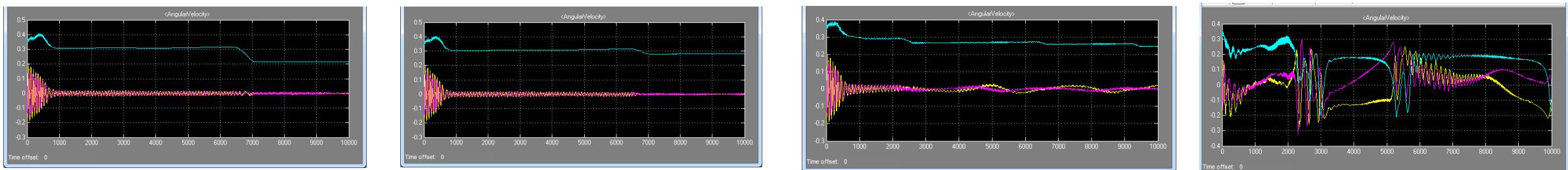


Figure 2 – Change of angular velocity of the spacecraft by $I = [0,0017; 0,0015; 0,0020] \text{ kg} \cdot \text{m}^2$

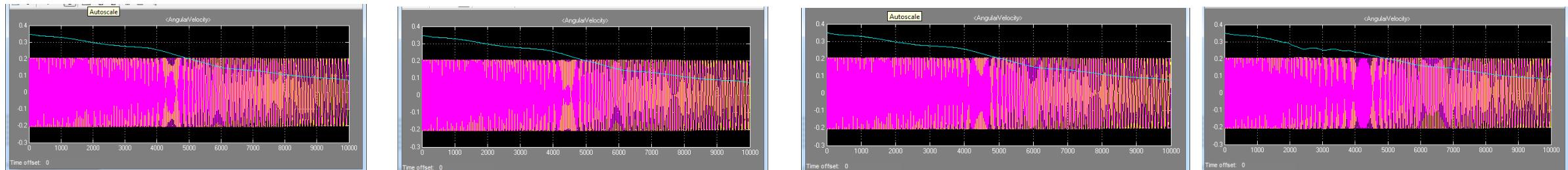
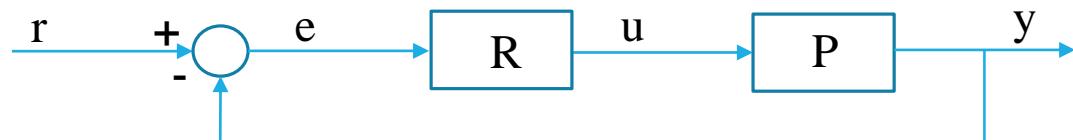


Figure 3 – Change of angular velocity of the spacecraft by $I = [0,4; 0,4; 0,08] \text{ kg} \cdot \text{m}^2$



$$\vec{m} = [m_x, m_y, m_z] \quad (17)$$

$$\vec{M} = \vec{m} \times \vec{B} \quad (18)$$

$$u(t) = K e(t) + T_d \frac{de(t)}{dt} \quad (19)$$

$$u_1 = -K_p^1 \Delta q_1 - K_d^1 \Delta \omega_1,$$

$$u_2 = -K_p^2 \Delta q_2 - K_d^2 \Delta \omega_2, \quad (20)$$

$$u_3 = -K_p^3 \Delta q_3 - K_d^3 \Delta \omega_3,$$

R — controller,

P — plant,

r — desired output,

e — tracking error, the difference between the desired output and the actual output

u — control signal,

y — actual output

K_d T_d - proportional gain, derivative gain

t — time,

$\Delta \omega_1, \Delta \omega_2, \Delta \omega_3$ - components of the vector of the deviation of the current angular velocity of the small spacecraft from the required one

$\Delta q_1, \Delta q_2, \Delta q_3$ - components of the vector part of the quaternion deviation of the current attitude of the small spacecraft from the required

Optimal approach based on the quadratic criterion quality

Approach based on the optimal location of the roots of the characteristic equation of a closed-loop control system

$$\frac{d\vec{X}}{dt} = \vec{AX} + \vec{Bu} \quad \vec{u} = K\vec{X} \quad (21)$$

$$J(\vec{u}) = \frac{1}{2} \int_0^\infty [\vec{\Delta X}^T W \vec{\Delta X} + \vec{u}^T P \vec{u}] dt \quad (22)$$

$\vec{\Delta X}$ - vector of deviation of the current state of the dynamic system from the required,

$\vec{\Delta X} = \vec{X} - \vec{X}_m$ \vec{u} - control vector; $W \geq 0, P > 0$ - constant matrices

$$\dot{\vec{h}}_1 + (A - BP^{-1}B^T R)\vec{h}_1 - W\vec{X}_m = 0 \quad K = P^{-1}B^T R.$$

$$W = diag\left(\frac{1}{\Delta q_1^{\max}}, \frac{1}{\Delta q_2^{\max}}, \frac{1}{\Delta q_3^{\max}}, \frac{1}{\Delta \omega_1^{\max}}, \frac{1}{\Delta \omega_2^{\max}}, \frac{1}{\Delta \omega_3^{\max}} \right)$$

$$\dot{R} + RA + A^T R - RBP^{-1}B^T R + W = 0$$

$$\vec{h}_1(t_k) = 0, R(t_k) = 0 \quad P = diag\left(\frac{1}{M_{1a}^{\max}}, \frac{1}{M_{2a}^{\max}}, \frac{1}{M_{3a}^{\max}} \right)$$

$\Delta \omega_i^{\max}, \Delta q_i^{\max}, i = \overline{1..3}$ - maximum deviations of the angular velocity and angular position of the small spacecraft

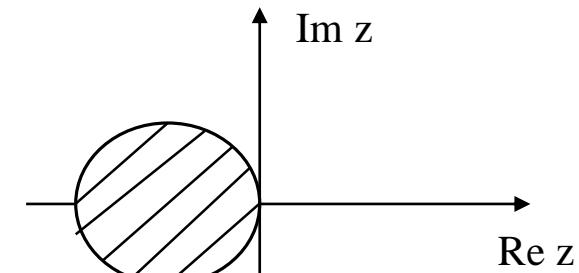
$M_{1a}^{\max}, M_{2a}^{\max}, M_{3a}^{\max}$, - maximum control moments of magnetic actuators.

$$K = ZY^{-1}$$

$$\begin{pmatrix} -rY & qY + AY + BZ \\ qY + YA^T + Z^T B^T & -rY \end{pmatrix} < 0, \quad Y > 0$$

$$A = \begin{bmatrix} 0 & 0 & 0 & \frac{1}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & \frac{1}{I_x} B_z^i & -\frac{1}{I_x} B_y^i \\ -\frac{1}{I_y} B_z^i & 0 & \frac{1}{I_y} B_x^i \\ \frac{1}{I_z} B_y^i & -\frac{1}{I_z} B_x^i & 0 \end{bmatrix}$$



$$\varphi_x = 60^\circ; \varphi_y = 100^\circ, \varphi_z = -100^\circ, \omega_x = -0.002 \text{deg/s}, \omega_y = 0.002 \text{deg/s}, \omega_z = 0.002 \text{deg/s}$$

$$\varphi_x = 0^\circ, \varphi_y = 0^\circ, \varphi_z = 0^\circ \quad \omega_x = 0.0 \text{deg/s}, \omega_y = 0.0 \text{deg/s}, \omega_z = 0.0 \text{deg/s}$$

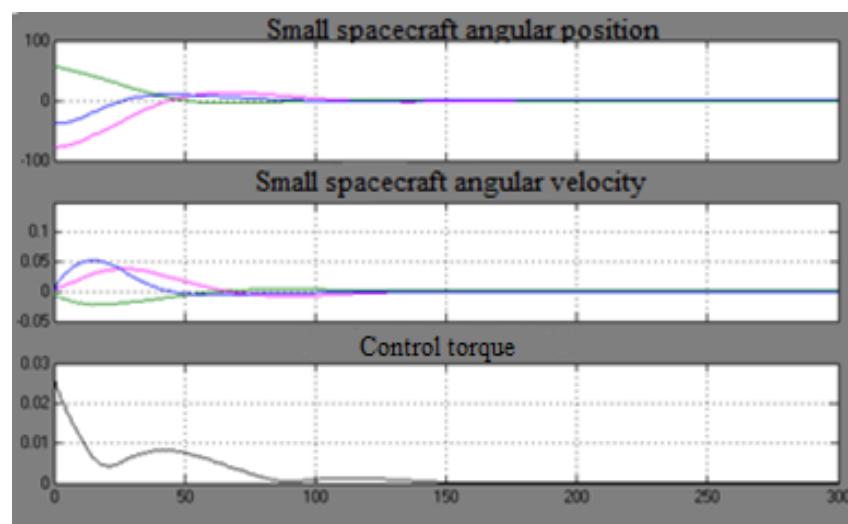


Figure 6. – Results of modeling the orbital orientation of the small spacecraft using a PD-controller to determine the magnetic moment when adjusting its coefficients using the optimal approach

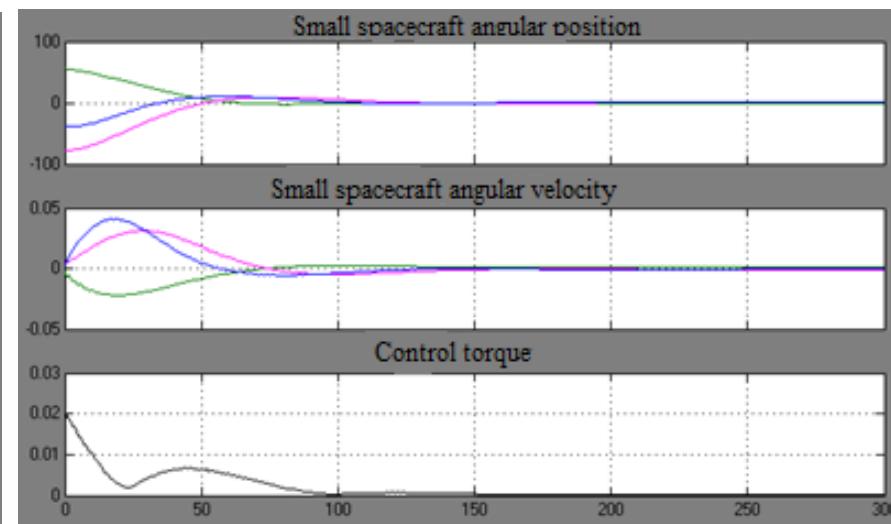


Figure 7. - Results of modeling the orbital orientation of the small spacecraft using a PD-controller to determine the magnetic moment when adjusting its coefficients by using the optimal location of the roots of the characteristic equation of a closed-loop control system

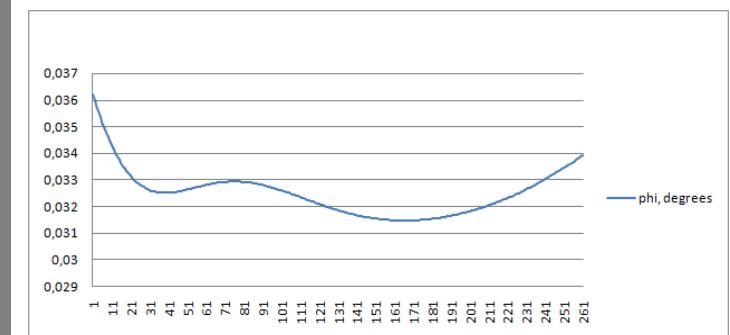


Figure 8 - The angular deviation from the direction to the Earth center of mass using a PD controller to determine the magnetic moment

$$\vec{S}(\vec{\Delta\omega}, \vec{\Delta Q}, t) = 0 \quad (23)$$

$$\vec{S} = \vec{\Delta\omega} + K_q \vec{\Delta q} \quad K_q > 0 \quad (24)$$

$$\vec{u} = \vec{u}_{\text{e}\kappa\theta} + \vec{u}_k \quad (25)$$

$$\vec{S} = 0, \dot{\vec{S}} = 0 \quad (27)$$

$$\vec{u} = \vec{u}_{\text{e}\kappa\theta} \quad (28)$$

$$\vec{u}_k = -\lambda \vec{S} \quad \lambda > 0 \quad (29)$$

$$\vec{u}_{\text{e}\kappa\theta} = [\vec{\omega}_{bi}^b \times] I \vec{\omega}_{bi}^b - \vec{M}_{dis} + I \vec{\omega}_T - \frac{1}{2} K_q I (\Delta q_0 E + [\vec{\Delta q} \times] (\vec{\omega}_{bi}^b - \vec{\omega}_T)). \quad (30)$$

$$\vec{m} = \frac{\vec{u} \times \vec{B}}{\|\vec{B}\|^2} \quad (31) \quad \vec{M}_a = \frac{\vec{m} \times \vec{B}}{\|\vec{B}\|^2} \times \vec{B} \quad (32)$$

$$\vec{S}^* = \vec{\Delta\omega} + K_q \vec{\Delta q} + \vec{A} e^{-at} \quad (33)$$

$$\vec{u} = \vec{u}_{\text{e}\kappa\theta}^* + \vec{u}_k^* \quad \vec{u}_k^* = -\lambda \vec{S}^* \quad (34)$$

$$\vec{u}_{\text{e}\kappa\theta}^* = [\vec{\omega}_{bi}^b \times] I \vec{\omega}_{bi}^b - \vec{M}_{dis} + I \vec{\omega}_T - \frac{1}{2} K_q I (\Delta q_0 E + [\vec{\Delta q} \times] (\vec{\omega}_{bi}^b - \vec{\omega}_T)) + a J A e^{-at} \quad (35)$$

$$\vec{A} = -\vec{\Delta\omega}(t_0) - K_q \vec{\Delta q}(t_0) \quad (36)$$

$$V = \vec{\Delta q}^T \vec{\Delta q} + (1 - \Delta q_0)^2$$

$$\dot{V} = -\vec{\Delta q}^T K_q \vec{\Delta q} < 0$$

$$\vec{\Delta q} = [0, 0, 0], \vec{\omega} = [0, 0, 0]$$

$$V = \frac{1}{2} \vec{S} (I \vec{S})$$

$$\dot{V} = -\vec{S} \lambda \vec{S} < 0$$

$$\varphi_x = 60^\circ; \varphi_y = 100^\circ, \varphi_z = -100^\circ, \omega_x = -0.002 \text{deg/s}, \omega_y = 0.002 \text{deg/s}, \omega_z = 0.002 \text{deg/s}$$

$$\varphi_x = 0^\circ, \varphi_y = 0^\circ, \varphi_z = 0^\circ \quad \omega_x = 0.0 \text{deg/s}, \omega_y = 0.0 \text{deg/s}, \omega_z = 0.0 \text{deg/s}$$

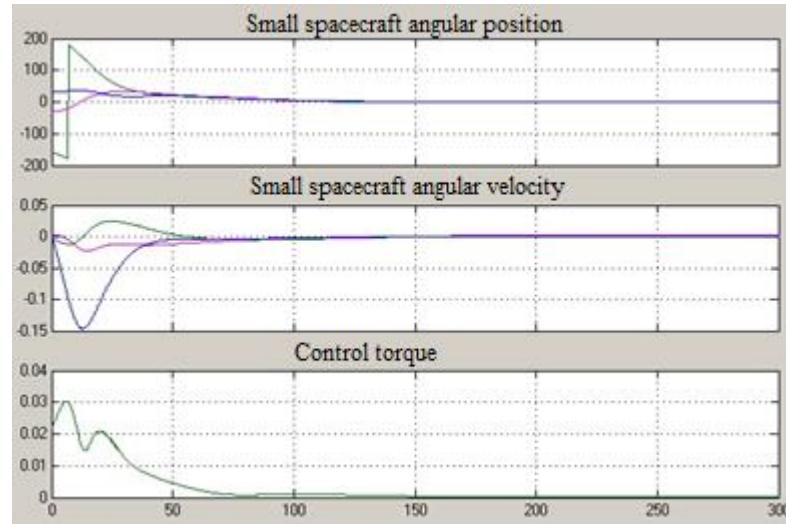


Figure 9. - Results of modeling the orbital orientation of the small spacecraft using the sliding mode controller to determine the magnetic moment

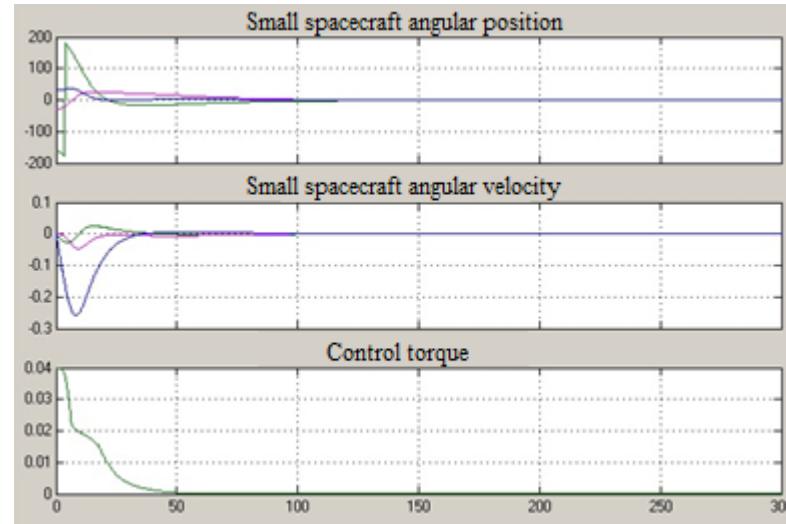


Figure 10. - Results of modeling the orbital orientation of the small spacecraft using the modified sliding mode controller to determine the magnetic moment

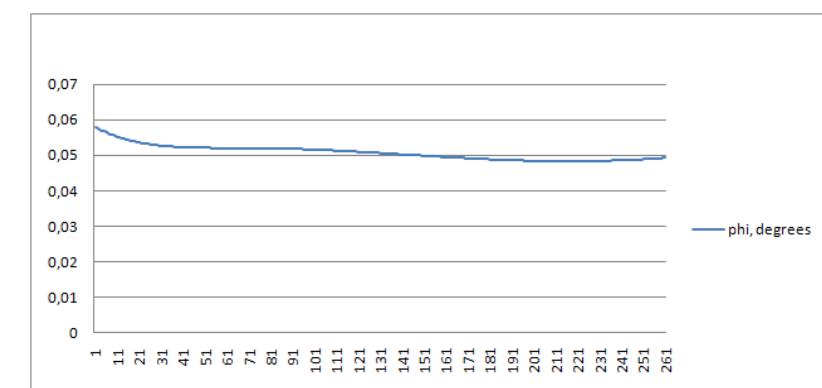


Figure 11. - Angular deviation from the nadir direction using the sliding mode controller to determine the magnetic moment

Quality of control in the orbital orientation mode

Methods of control	Duration of inertial orientation maneuver, s	Orientation accuracy (module of the vector part of the orientation quaternion)
PD control	150	0.0044
Sliding mode control	150	0.0088
Modified sliding mode control	150	0.0048

Quality of control in the maintaining orbital orientation mode

Methods of control	Duration of inertial orientation maneuver, s	Orientation accuracy (module of the vector part of the orientation quaternion)
PD control	260	0.004
Sliding mode control	260	0.01

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Modeling and Optimization in Space Engineering

State of the Art and New Challenges

 Springer

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