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INTRODUCTION

The main task of geophysics is to study the geological structure of the earth's crust. One of its main sections is electrical prospecting. Methods of electrical prospecting have found wide application in geological research and exploration of mineral deposits. Studying the parameters of the electromagnetic field that arises under the influence of sources of direct and alternating current, we can judge the structure of the geological section.

Now in geoelectrics there are more than fifty methods and modifications used in both in-depth studies and in studying the upper part of the section. The effectiveness of reconnaissance depends on the internal structure of the medium. When geological work with the surface part of the earth's crust, it is more often necessary to deal with sedimentary mountain peoples. They cover more than 70% of the continents and most of the minerals are associated with them. These natures form strata lying horizontally in the earth's crust or in the form of complex folds. In the future, we will use a horizontally layered model of the earth. (Fig 1)

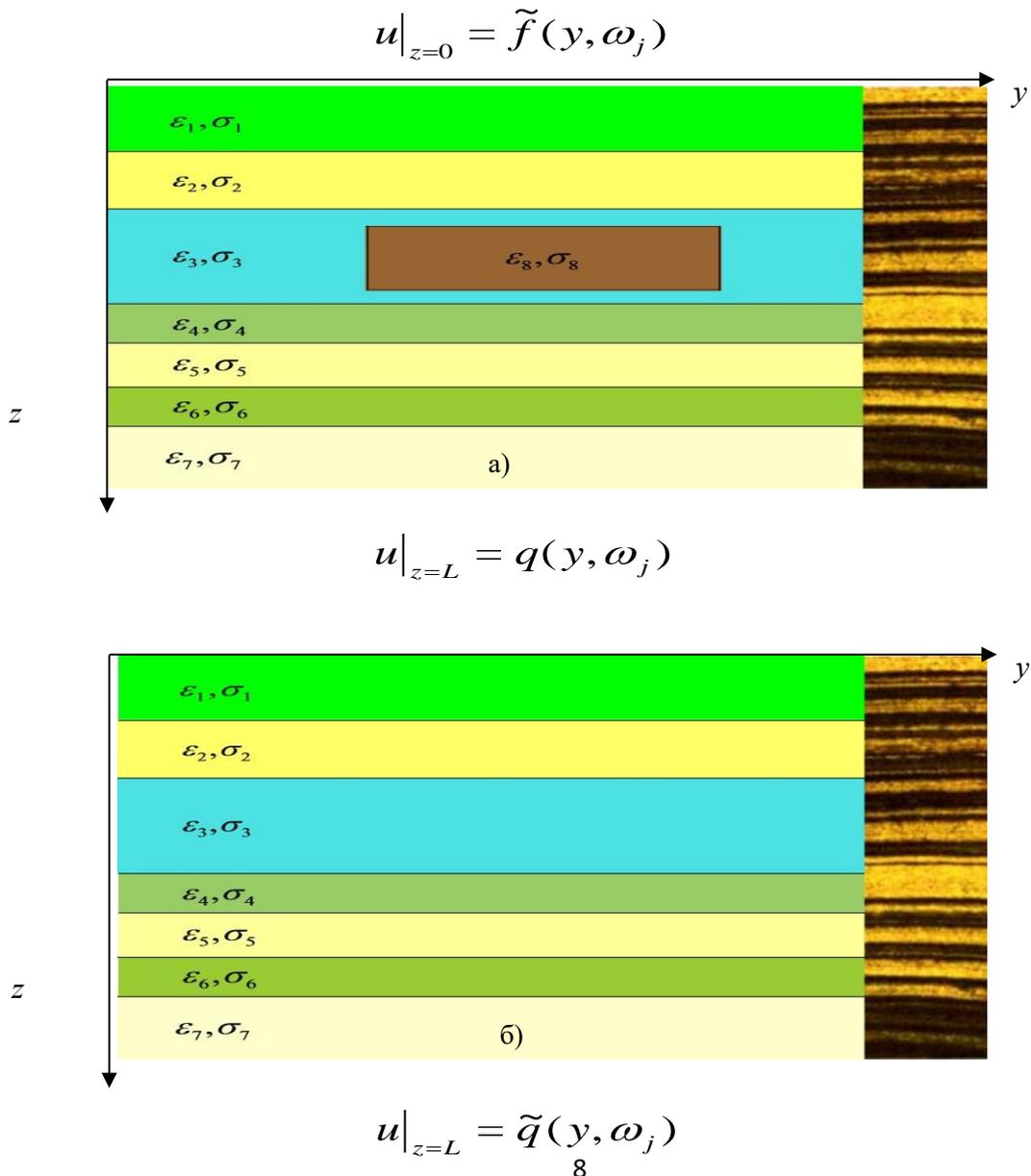


Figure 1. Model of a horizontally layered medium.
a) with heterogeneity,
b) homogeneous medium

Knowing the structure and properties of the medium, we can, by specifying the initial conditions and the source, predict how the alternating electromagnetic field behaves. In particular, we can calculate how it will manifest itself on the surface of the earth.

In the thesis the problems concerning the justification and development of problems of continuation of the electromagnetic field to a given depth are considered. They are of fundamental importance in the research of information about the characteristics of the Earth's structure.

Of particular value in calculations are georadar data, which, in a comparative analysis with the task of extending the electromagnetic field to a depth of h , allow us to determine with greater accuracy the number of inhomogeneities.

Georadar is one of the most common devices for electromagnetic sensing. This device is widely used for soil survey and for the control of building structures. Each GPR includes a source and a receiver. A high-frequency electromagnetic signal is generated in the source, which, penetrating into the medium, is reflected from the separation line of the layers and fixed by the receiver. Further, the obtained data is analyzed and a conclusion is made about the properties of the medium under study.

Currently, most radars can be grouped into several types, which differ in the basic principles of operation: stroboscopic georadars, weak pulse pulses, super-powerful radars with spaced antennas.

Georadars of the "LOZA" series belong to the class of superpower radars (Table 1) with diversity antennas for studying the subsurface soil structure to depths of up to hundreds of meters, depending on the model of the instrument, the antenna used and the parameters of the sounded medium

Table 1 – Georadar parameters

№	Georadars name	Power (W)	Maximum depth of sounding, (m)
1	Scorpio	7	3-5
2	U-explorer	7	3-5
3	OKO	60	24
4	Nemfis	600	до 10
5	Loza	1 000 000	до 100

The capabilities of radars in the depth of sounding are characterized by the magnitude of the real energy potential, which is numerically equal to the maximum attenuation of the probe pulse in the medium at which the device is able to detect the object. For many georadars, the value of the real energy potential varies from 20 to

40 dB. This means that when the signal attenuates 10 to 100 times, the device will not be able to detect objects. The Loza-V radar has a large energy potential (not less than 120 dB), which allows to work in environments with high conductivity, for example, in loam or wet clay.

The operation of the georadars of the Loza series showed their effectiveness in solving the following problems:

- Engineering and geological surveys for the design of linear and area structures;
- Monitoring of the state of soil in the area of engineering structures during construction and operation (foundations of buildings, railways and highways);
- Detection of underground metal and non-metallic communications;
- Search in the ground and in building structures of cavities of artificial and natural origin (karst cavities, voids and tectonic disturbances);
- Determination of the thickness of loose and deformed soils, mapping of the boundaries of bedrock under loose sediments;
- Search for sand and gravel sediments, mapping of landslide bodies;
- Determination of the state of bridges, supports, tunnels;
- Detection of disposal of environmentally hazardous waste;
- Non-destructive mapping of closed containers, hidden trenches, boundaries of contaminated soils, leaks.

Relevance of the research topic. Currently, the use of non-destructive methods to identify the internal structure and properties of hidden subsurface objects can significantly reduce the costs of searching for or diagnosing the state of the objects under investigation.

The thesis is devoted to the study of the inverse problem of geoelectrics, the development of a numerical method for its solution.

The use of georadars in sounded environments requires highly qualified specialists to interpret the obtained data. In this regard, georadar technologies cannot be widely used in practice. The development of the mathematical apparatus for solving inverse problems of electromagnetic sounding can overcome the above problem, minimize the participation of specialists in achieving the final result and obtain the maximum information from georadar data. Such problems pertain to inverse and ill-posed problems, the basic theories of which were laid down in the works of A.N. Tikhonova, M.M. Lavrentyeva, V.K. Ivanova, VG Romanova. A great contribution to the development of the theory and numerical methods for solving inverse and ill-posed problems was made by the Kazakh mathematicians - S.Elubaev [1], SITemirbulatov [2], U.M. Sultangazin [3], S.Ya. Serovaisky [4] , E.Y. Bidaibekov [5], M.O. Otelbaev [6], A.Sakabekov [7], SAAtanbaev [8-9], BEKanguzhin [10], B.Rysbayuly [11], KT Iskakov [12] G.B Bakhanov [13], NT Danayev [14], T. Sh. Kalmenov [15-16], ST Mukhambetzhanov [17], TS Kenzhebaev [18], MA Bektemesov [19-20], AS Berdyshev, S.S.Bimuratov, D.B. Nurseitov [21], A.Nurseitova [22], as well as many others.

The problems of continuation of solutions of equations of mathematical physics from a part of the boundary in many cases are strongly ill-posed problems in classes of functions of finite smoothness. In the thesis the approach of regularization of the

field continuation problem is proposed. For the first time, a similar iterative method was proposed in the work of V.A. Kozlova, V.G. Maz'ya, A.V. Fomin in 1991 [23].

The inverse problem of electrical prospecting is to determine the electromagnetic parameters of the Earth, from the electromagnetic field observed on the surface. This process is usually called the interpretation of the results of field observations. The transition from the observed field to the real electromagnetic medium is complicated by some ambiguity in the solution of the inverse problem, since electromagnetic media with different values of parameters can correspond to a given electromagnetic field obtained as a result of observations with practically attainable accuracy. From these environments, the interpreter must choose one that corresponds to the true geological structure of the region under study. To reduce the degree of ambiguity in the solution of the inverse problem, i.e. to increase the reliability of the interpretation, it is necessary to use previously obtained data on the electrical properties of the medium, data on its geological structure and the results of other geophysical studies.

Incorrect problems of continuation from the part of the boundary of electromagnetic fields towards the occurrence of inhomogeneities, as well as inverse problems of determining the coefficients of the equations of electrodynamics, are actual and practically important.

The purpose and objectives of the study. The aim of the work is to develop, as well as software implementation of the numerical method of one ill-posed problem for the two-dimensional equation of electrodynamics.

In accordance with the goal set in the work, the following research objectives are formulated:

- Investigation of the problems of continuation of the solution from the part of the boundary of the equation of electrodynamics;
- Development of a numerical method for solving the problem under study;
- Development of a computational algorithm for solving the problem being solved;
- Analyze the numerical solution in the presence of several localized objects;
- Comparative analysis between experimental and computational data for different deposits;

The practical significance of the results. The practical importance of the dissertation work is determined by the possibility of changes in the developed algorithms and programs for diagnosing submarine and underground objects with a high degree of accuracy, determines their quantities. These calculations contribute not only to diagnose with a high degree of accuracy underwater and underground objects, but also to determine their number. The developed algorithms facilitate and improve methods for determining the state of localized objects (roads, dams, runways, archaeological burial sites, construction sites) to create the best opportunities for comparative analysis of calculations of georadar data

Novelty of the work. Scientific novelty consists in developing a numerical method for solving the problem of extending the electromagnetic field to a given depth.

In the present paper the following results were obtained:

- The problem of continuation of the solution from a part of the boundary of the electrostatics equation is investigated;
- developed a numerical method for solving the problem;
- developed a computational algorithm for solving the problem being solved;
- analysis of numerical solution in the presence of several localized objects;
- Comparative analysis between experimental and computational data for different deposits is carried out;

Approbation of the results of the dissertation. Sholpanbaev B.B , Dairbaeva L.M , Askarova Z.B. The problem of continuation of the electromagnetic field to a given depth// Abstracts of the International Conference “Inverse Problems in Finance, Economics and Life Sciences”, Almaty, Kazakhstan, December 26-28, 2017.

The scope and content of dissertation work. The dissertation work consists of introduction, three chapters, conclusion and bibliography. The dissertation work contains 60 pages, 18 figures, 1 tables, bibliography consisting of 28 items.

1 MATHEMATICAL MODEL OF THE EQUATION OF ELECTRODYNAMICS

1.1 Sounding

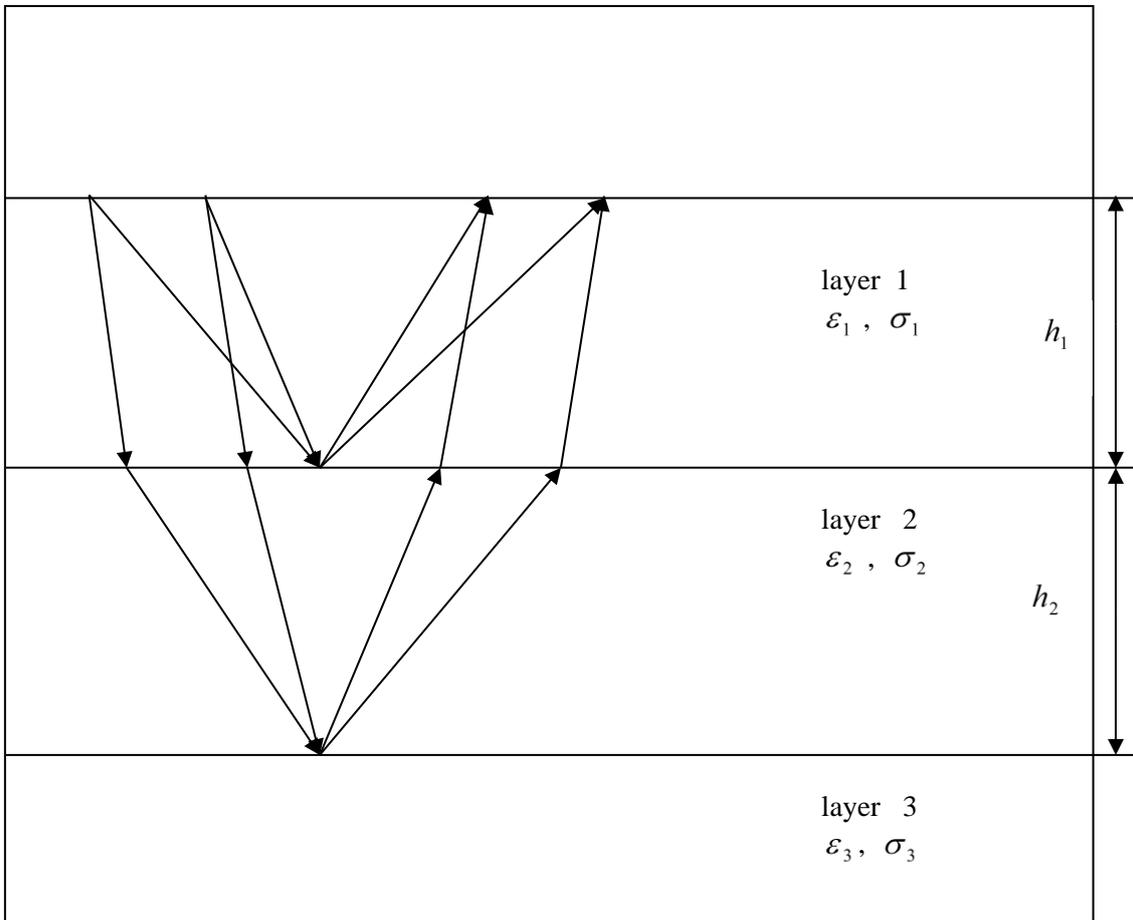


Fig. 2 Scheme of sounding

(Initially, the transmitter and receiver are at position 1, then at position 2).

It can be seen from the above diagram that the number of independent equations is doubled during the sounding. Increasing the separation points more than two does not increase the number of independent equations, however, when probing, more points are usually used than two. This is due to the following reasons: firstly, a large number of points make it possible to better identify the travel time; secondly, the accuracy of calculations increases. Here, in principle, it is possible to obtain the accuracy of estimates higher than the step of discretizing the corresponding parameter.

Let us write below the equation for the arrival time and the amplitude of the pulse reflected from the boundary between the first and second layer.

$$t_1 = \frac{2l_1\sqrt{\varepsilon_1}}{c} \quad (1.1.1)$$

$$t_2 = \frac{2l_2\sqrt{\varepsilon_1}}{c} \quad (1.1.2)$$

$$A_1 = A_0 R_{12}(\alpha) \exp(-2p_1 l_1) \quad (1.1.3)$$

$$A_2 = A_0 R_{12}(\alpha) \exp(-2p_1 l_2) \quad (1.1.4)$$

Here l is the geometric path length of the pulse in the medium from the radiation point to the reflection point, equal to the path from the reflection point to the receiving point

$$l = \sqrt{h^2 + \frac{d^2}{4}} \quad (1.1.5)$$

It is determined by the Pythagorean theorem through an unknown layer thickness h and a known distance between the transmitter and the receiver d . The reflection coefficient $R(\alpha)$ for oblique incidence depends on the angle of incidence α . It can be taken into account through Snell's law, or, if $h \gg d$, accept $R(\alpha) \approx R(0)$.

Equations (1.1.1-1.1.5) for the first pulse make it possible to determine for the first layer all the parameters: εl , σl , $h l$, and also the permittivity of the second layer ε_2 , through the reflection coefficient R_{12} . It is obvious that analogous equations can also be written for all subsequent reflected pulses. Thus, the sounding method makes it possible to uniquely solve the inverse problem of georadiolocation in a plane-layered formulation.

When processing the sounding data, only equations (1.1.1-1.1.5) are most often used for the delay time, which determine the travel time. From these equations, it is possible to obtain the true depth of the layer $h l$ and the permittivity εl . Most often, this data is sufficient to solve most practical problems.

As already mentioned, the hodograph is the dependence of the delay time t on the distance d between the receiver and the transmitter. It is given by the formula:

$$t = 2 \frac{\sqrt{\varepsilon}}{c} \sqrt{h^2 + \frac{d^2}{4}} \quad (1.1.6)$$

Formula (1.1.6) specifies the time curve for absolute synchronization, when the receiver starts counting down the time immediately after the pulse radiation. In our georadars, synchronization is carried out by the air wave from the probing pulse of the transmitter, so the start of registration lags behind the radiation time by $t' = d/c$.

$$t = 2 \frac{\sqrt{\varepsilon}}{c} \sqrt{h^2 + \frac{d^2}{4}} - \frac{d}{c} \quad (1.1.7)$$

If the function of the classical hodograph (1.1.6) always increases with increasing d , then in the georadar for small d the delay time always decreases, passes through a minimum, and only then begins to increase.

Two measurements of signal delays t_1 and t_2 at distances between the receiver and transmitter d_1 and d_2 according to the hodograph equation (1.1.6) uniquely determine the thickness of the layer h_1 and its dielectric permittivity ε_1 according to the following formulas:

$$h_1 = \sqrt{\frac{t_2^2 d_1^2 - t_1^2 d_2^2}{4(t_1^2 - t_2^2)}} \quad (1.1.8)$$

$$\varepsilon_1 = \frac{t_1^2 c^2}{4h_1^2 + d_1^2} \quad (1.1.9)$$

The design of low-frequency georadars makes it possible to distribute the transmitter and receiver antennas for distances of up to tens of meters without loss of synchronization, which makes it possible to record and identify travel curves with high accuracy.

Most domestic and foreign georadars have permanent antenna designs, which exclude the possibility of conducting sounding - the most informative georadar survey method, which uniquely solves the inverse problem. This probably explains the widespread opinion that georadar data cannot be interpreted without drilling data.

Modifications of georadars that do not have registration of the full wave form fix only the exceeding of one or three thresholds by the signal (binary registration by adjustable thresholds). The lack of information about the amplitude makes it possible to determine only the thicknesses of the layers and their dielectric permittivity when probing. The characteristics of the lower layer lying behind the last fixed boundary are not determined.

When registering the waveform, it therefore becomes possible, in addition, to determine the conductivity of each layer above the last boundary and the dielectric constant of the layer beyond the last boundary.

With binary registration, most often in low-impedance media, situations arise when a low-frequency "tail" appears from the upper boundaries, which outputs weak signals from the threshold zone, and they do not intersect it.

Determination of the depth of occurrence of localized objects.

When profiling, it is possible to determine the depth of occurrence of individual local objects and the dielectric constant of the medium covering them, because of the rather wide directional pattern of the transmitting and receiving antenna. When traveling along a route, a separate local object is observed not only at the point directly above it, but also at some distance to both sides, the magnitude of this removal being determined by the angular width of the antenna pattern. In Fig. 3 shows the pulse propagation pattern for the single object profiling.

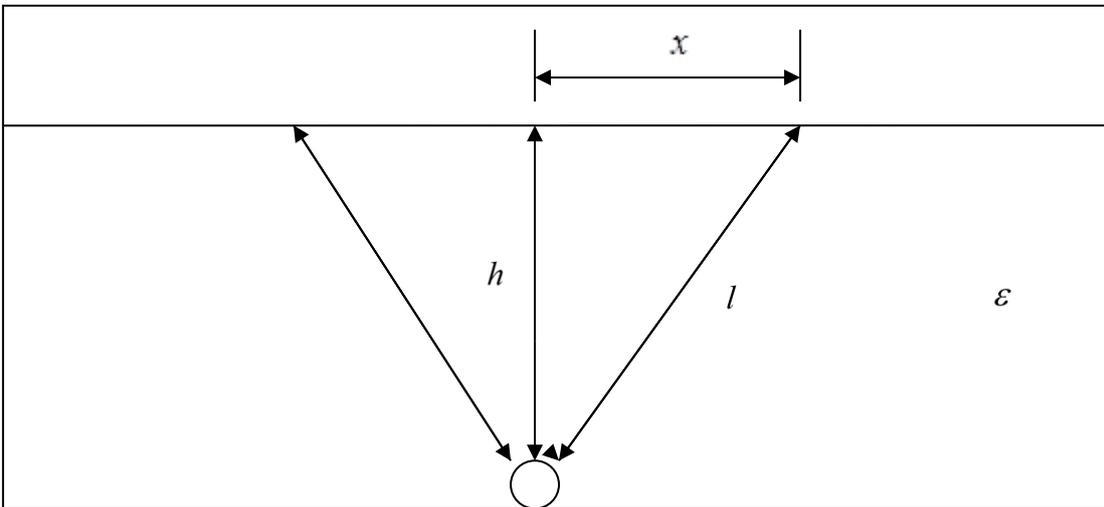


Fig.3. Trajectories of pulse propagation during the profiling of single objects.

(Because of the finite width of the radiation pattern, the object is observed in a certain zone from the point of occurrence).

The delay time t is determined from the geometry shown in Fig. 3.

$$t = 2l \frac{\sqrt{\varepsilon}}{c} = \frac{2\sqrt{\varepsilon}}{c} \sqrt{h^2 + x^2} \quad (1.1.10)$$

The formula (1.1.10) has the same form as the hodograph formula, so from it, from two measurements of the delay time at two points, it is possible to calculate the depth of the object and the dielectric constant of the covering medium. Let t_0 be the

delay time of the signal above the object, t_1 is the delay time at the distance x_1 , then the depth and permittivity are defined as

$$h = \sqrt{\frac{t_0^2 x_1^2}{t_1^2 - t_0^2}} \quad (1.1.11)$$

$$\varepsilon = \frac{t_0^2 c^2}{4h^2} \quad (1.1.12)$$

As a single object, pipes and cable most often act. But it should be remembered that these objects can be considered single and use formulas (1.1.11-25) only when perpendicular to their intersection with the profile. The intersection at any other angle violates the geometry of the problem (Fig.3) and leads to incorrect results.

In this method of determining the depth and the permittivity of using the signal amplitude, one can not speak, since the estimation of the reflection coefficient from a single object is in itself a difficult task, even if its exact structure is known.

So, from the above we can draw the following conclusions:

- the most informative method of georadar survey is sounding. For a plane-layered medium, this method allows one to determine all the parameters of the medium: the thicknesses and depths of layers, their dielectric permeability and conductivity.
- the profiling method does not allow unambiguous identification of any parameters of the environment without additional assumptions or without additional data.
- the most optimal method for time-consuming georadar surveying is the initial profiling of the route and the determination of the sounding points. These points are chosen on the plane-layered sections of the route. If no new layers appear on the remaining route on the remaining route, then one probe point is sufficient. The data obtained during the sounding are then used as additional data for profiling, which allows solving the inverse problem on the entire profiling path.
- making a conclusion about the ambiguity of the solution of the inverse problem in the profiling, we considered the process of pulse propagation in the high-frequency asymptotics, when there are no dispersion distortions of their shape. In the low-frequency region, apart from the damping of the pulse, dispersion distortions will be observed. It is possible that the laws of changing the shape of the pulse can be used as additional information for solving the inverse problem in the low-frequency region.

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The profiling method does not allow unambiguous identification of any parameters of the medium without additional assumptions or without additional data.

The most optimal way to conduct georadar survey is to initial the route profiling and to determine the sounding points. These points are chosen on the plane-layered sections of the route. If no new layers appear on the remaining route on the remaining route, then one probe point is sufficient. The data obtained during the sounding are then used as additional data for profiling, which allows solving the inverse problem on the entire profiling path.

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1.2 The Maxwell Equations

The formulation of the Maxwell equations was preceded by the discovery of the laws of interactions of charged bodies, magnetized bodies and conductors with an electric current. At the same time, the Coulomb law, the Ampere law, the Bio Savar-Laplace laws were formulated.

In 1831 M. Faraday discovered the phenomenon of electromagnetic induction and formulated a law describing this phenomenon. In addition, around the same time he introduced the concept of the electric and magnetic fields as independent physical substances.

Based on the Faraday representation about the field, and introducing a bias current, equal in its magnetic effect to the usual electric current, JC Maxwell formulated a system of equations, called after the Maxwell equations.

Maxwell's equations functionally connect the electric and magnetic fields with charges and currents and cover all the known laws of electromagnetism. Initially, Maxwell resorted to auxiliary mechanical models of the "ether", but already in the Treatise on Electricity and Magnetism (1873), the electromagnetic field was regarded as an independent physical object.

The physical basis of Maxwell's equations is the short-range principle, which states that the transmission of electromagnetic perturbations from point to point occurs at a finite rate (in a vacuum at the speed of light c). This principle is opposed to the classical Newtonian principle of long-range action, which reduces to instantaneous transmission of impacts to any distance.

The mathematical apparatus of Maxwell's theory is vector analysis. Maxwell believed that his merit was only in the mathematical formulation of Faraday's ideas.

The canonical form

The canonical form of the record, now accepted, belongs to H. Hertz and O. Heaviside and is based on the use of vector fields: the electric field strength, the magnetic field strength, the electric induction vectors and the magnetic induction. Maxwell's equations relate these quantities to each other, as well as to the density of electric charge and the density of electric current, which are considered as sources of electric and magnetic fields, respectively.

The first Maxwell equation

In 1820, the Danish physicist Oersted demonstrated electric current and discovered that there is a magnetic field around the conductor with current. This was detected by the action of electric current on the magnetic arrows located near the conductor with current. Experimentally, using iron filings or a set of magnetic arrows, it was found that the magnetic lines are closed. Then we can say that the magnetic field has a vortex character. Mathematically, this is written using the "rotor" operator, this operator is written with the *rot* or $\vec{\nabla}$ symbols. The characteristic of the magnetic field is its strength \vec{H} , so we speak of magnetic field vortices. They exist not only around the conduction current, but also around the bias current. Then this experimental fact can be written using the equation:

$$\text{rot}\vec{H} = \vec{j}_{npob} + \vec{j}_{cm} \quad (1.1)$$

$$\left[\vec{\nabla} \cdot \vec{H} \right] = \vec{j}_{npob} + \vec{j}_{cm} \quad (1.2)$$

Here \vec{j}_{npob} is the conductivity current density, and \vec{j}_{cm} is the displacement current density.

Formula (1.1) is the first Maxwell equation in differential form. In interpreting this equation, it is necessary to understand the following:

1. Equation (1.1) states that there is a vortex magnetic field around any current;
2. The expression "electric current generates a magnetic field" is not entirely correct for a direct current, since there is no reference system in which a conductor with current would exist separately from the magnetic field;
3. One cannot say that a constant current generates a constant magnetic field, since they exist in unity, and there is no cause-effect relationship;
4. In the case of variable fields, it can be said that a changing electric field generates a magnetic field. But this is discussed in another Maxwell equation;
5. Equation (1.1) is a description of an infinitesimal neighborhood of some point under study.

We get the first Maxwell equation in integral form. To do this, we multiply the scalar formula (1.1) by a vector $d\vec{S}$ and integrate over the surface of the area:

$$\int_S \text{rot}\vec{H} \cdot d\vec{S} = \int_S \vec{j}_{npob} \cdot d\vec{S} + \int_S \vec{j}_{cm} \cdot d\vec{S} \quad (1.3)$$

We apply the Stokes formula to the left-hand side of equation (1.3):

$$\int_S \text{rot} \vec{H} \cdot d\vec{S} = \oint_L \vec{H} \cdot d\vec{l} = \oint_L H_l \cdot dl \quad (1.4)$$

Here L is a closed contour bounding the surface S , and H_l is the projection of the vector of the magnetic field intensity on the tangent to the contour. We substitute the formula (1.4) into the formula (1.3):

$$\oint_L \vec{H} \cdot d\vec{l} = I_{npob} + I_{cm} \quad (1.5)$$

Here, by definition of the current density, the values of the conduction current and displacement current are written:

$$\int_S \vec{j}_{npob} \cdot d\vec{S} = I_{npob} \quad (1.6)$$

$$\int_S \vec{j}_{cm} \cdot d\vec{S} = I_{cm} \quad (1.7)$$

Equation (1.5) is the first Maxwell equation in integral form. It has the same meaning as the equation in differential form, only here we are talking about a finite closed loop and a finite area.

The physical essence of the first Maxwell equation in integral form - around the conduction current and displacement current, there are vortices of the magnetic field.

The second Maxwell equation

The second Maxwell equation is a mathematical formulation of Faraday's law for the phenomenon of electromagnetic induction. This phenomenon was discovered in 1831. The essence of the phenomenon of electromagnetic induction is that when the magnetic field changes in a closed conductor in this field, an electric current arises.

In order to create an electric current in the conductor, it is necessary to have an electric current source, an EMF source. As is known, the EMF is provided by external forces, that is, forces whose closed-loop operation is not zero. In a closed conductor in an alternating magnetic field, electric charges move into motion, therefore, electric forces act on them, but not electrostatic forces. Such forces are forces acting from the side of a new vortex electric field, which arises in the presence of an alternating magnetic field.

Since a conductor in which an electric current is generated in an alternating magnetic field is most often associated with a certain material medium, then the concept of magnetic field induction is used to quantify the magnetic field \vec{B} . The induction of the magnetic field is related to the strength of the magnetic field \vec{H} in the system of SI units by the formula:

$$\vec{B} = \mu_0 \cdot \mu \cdot \vec{H} \quad (1.8)$$

Here $\mu_0 = 4\pi \cdot 10^{-7} \frac{\Gamma_H}{M}$ is the magnetic constant of vacuum, which is a fundamental constant, and μ is the magnetic permeability of the medium, which shows how many times the intensity of the magnetic field μ in the substance is greater than the intensity of the magnetic field in vacuum.

As a generalization of experience, Faraday's law for the phenomenon of electromagnetic induction has the form:

$$E_{ind} = - \frac{d\Phi}{dt} \quad (1.9)$$

Here - E_{ind} EMF induction, Φ - magnetic flux. The magnetic flux is determined by the following formula:

$$\Phi = \int_S \vec{B} \cdot d\vec{S} = \int_S B \cdot dS \cdot \cos(\vec{n}, \vec{B}) = \int_S B_n \cdot dS \quad (1.10)$$

Here, \vec{n} the vector of the normal to the surface is an area S , B_n is the projection of the magnetic field induction vector onto the normal vector to the site.

The minus sign in formula (1.9) describes the Lenz rule, which is based on the law of conservation of energy. According to the Lenz rule, when a magnetic flux penetrating a circuit changes, an induction current appears in it, this by its magnetic field hinders the cause of its calling.

If there is an alternating magnetic field in space, but there is no conductor, then a vortex electric field will appear in space, which will manifest itself as an EMF when the conductor is placed in such an alternating magnetic field.

Then, generalizing these experimental facts, we can write the second Maxwell equation in a differential form:

$$rot \vec{E} = - \frac{\partial \vec{B}}{\partial t} \quad (1.10.1)$$

$$\left[\vec{\nabla} \cdot \vec{E} \right] = -\frac{\partial \vec{B}}{\partial t} \quad (1.10.2)$$

We multiply the scalar expression (1.10.1) by $d\vec{S}$ and integrate over the entire surface area covered by some mentally introduced conductor:

$$\int_S \text{rot} \vec{E} \cdot d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} \quad (1.11)$$

The left-hand side of (2.11) is transformed according to the Stokes formula:

$$\int_S \text{rot} \vec{E} \cdot d\vec{S} = \oint_L \vec{E} \cdot d\vec{l} = \oint_L E_l \cdot dl = E_{uho} \quad (1.12)$$

Formula (1.12) determines the work of external forces on the displacement of a single positive charge along a closed circuit, which from the course of elementary physics is called the electromotive force of a current source. Here the role of the current source is played by a time-varying magnetic field.

We substitute the formula (1.12) into the formula (1.11) and take into account that integration and differentiation can be interchanged:

$$E_{uho} = -\int_S \frac{\partial \vec{B}}{\partial t} \cdot d\vec{S} = -\frac{\partial}{\partial t} \int_S B_n \cdot dS \quad (1.13)$$

The third Maxwell equation

In order to obtain the third Maxwell equation, we use the second Maxwell equation in the differential form:

$$\text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1.14)$$

We perform the divergence operation with the left and right sides of equation (1.14):

$$\text{div} \text{rot} \vec{E} = -\text{div} \frac{\partial \vec{B}}{\partial t} \quad (1.15)$$

We consider the differential operators on the right-hand side of (1.15):

$$\operatorname{divrot}\vec{E} = \vec{\nabla} \left(\left[\vec{\nabla} \cdot \vec{E} \right] \right) = \vec{E} \left(\left[\vec{\nabla} \cdot \vec{\nabla} \right] \right) = 0 \quad (1.16)$$

When writing formula (1.15), the properties of the mixed product of vectors are used, and that the vector product of identical vectors is zero.

We substitute the formula (1.16) into the formula (1.15) and get:

$$\operatorname{div} \frac{\partial \vec{B}}{\partial t} = 0 \quad (1.17)$$

Since the divergence is an operation of differentiation with respect to coordinates, which, like time, are independent variable quantities, the operations of divergence and time derivative can be interchanged:

$$\frac{\partial}{\partial t} \operatorname{div} \vec{B} = 0 \quad (1.18)$$

It follows from (1.18) that:

$$\operatorname{div} \vec{B} = \operatorname{const} \quad (1.19)$$

The value *const* in formula (1.19) is determined from the first conditions. For first conditions, we can choose points of space in which there is no magnetic field at the first instant of time. Then the value *const* will be zero:

$$\operatorname{const} = 0 \quad (1.20)$$

We substitute the formula (1.20) into the formula (1.19) and obtain the third Maxwell equation, which has the form:

$$\operatorname{div} \vec{B} = 0 \quad (1.21)$$

The third Maxwell equation (1.21) states that within a neighborhood of a point in a magnetic field, the magnetic field vector does not break. In other words, the lines of the magnetic field induction vector have neither a beginning nor an end, they are closed.

We get the integral form of writing the third Maxwell equation. To do this, we multiply expression (1.21) by an element of volume dV and integrate over the entire volume:

$$\int_V \operatorname{div} \vec{B} \cdot dV = 0 \quad (1.22)$$

Now we use the Gauss theorem, according to which we obtain:

$$\int_V \operatorname{div} \vec{B} \cdot dV = \oint_S \vec{B} \cdot d\vec{S} = \oint_S B_n \cdot dS \quad (1.23)$$

In (1.23), the closed surface S bounding the selected volume is the projection of the induction vector B_n of the magnetic field onto the normal to the closed surface.

Combining the results of formulas (1.22) and (1.23), we get the third Maxwell equation in integral form:

$$\begin{aligned} \oint_S \vec{B} \cdot d\vec{S} &= 0 \\ \oint_S B_n \cdot dS &= 0 \end{aligned} \quad (1.24)$$

The integral form of the third Maxwell equation has the same meaning as the differential form, but it refers not to a point, but to a certain finite volume. In fact, this equation shows: how many lines of magnetic field induction "comes" into some allocated volume, the same amount "out" of this volume. The lines of the magnetic field induction vector do not begin anywhere and do not end anywhere. They are closed.

Here, instead of the concept of "lines of force", the concept of the vector induction line or the tension vector is used. This is a more precise concept, especially if we consider a magnetic field in which the induction line is determined not by force, but by the moment of force.

The third Maxwell equation can be written as a generalization of the experimental fact that the lines of induction of the magnetic field are closed. For this it is enough to record this fact using vector analysis.

The fourth Maxwell equation

The fourth Maxwell equation was historically derived by Gauss and is known as the Gauss theorem or the Ostrogradsky-Gauss theorem. Therefore, it can be in two ways.

The first way to derive the fourth Maxwell equation uses Maxwell's first equation, and in this case is preferable, since it checks the systemic nature of these equations. The essence of this conclusion is as follows.

We write the first Maxwell equation:

$$\operatorname{rot} \vec{H} = \vec{j}_{npob} + \vec{j}_{cm} \quad (1.25)$$

We use the definition of the bias current density:

$$\vec{j}_{cm} = \frac{\partial \vec{D}}{\partial t} \quad (1.26)$$

Here \vec{D} is the vector of electrostatic displacement or vector of electrical induction.

We substitute the formula (1.26) in the formula (1.25) and carry out the operation "divergence" with respect to the right and left parts:

$$\text{divrot}\vec{H} = \text{div}\vec{j}_{npob} + \text{div}\frac{\partial \vec{D}}{\partial t} \quad (1.27)$$

As was shown in the derivation of the third Maxwell equation. The left-hand side of expression (1.27) is zero. Then we interchange the differentiation with respect to time and with respect to the coordinates on the right-hand side of (1.27), we get:

$$0 = \text{div}\vec{j}_{npob} + \frac{\partial}{\partial t} \text{div}\vec{D} \quad (1.28)$$

We use the law of conservation of electric charge in vector form:

$$\text{div}\vec{j}_{npob} = -\frac{\partial \rho}{\partial t} \quad (1.29)$$

Here $\rho = \frac{\partial q}{\partial V}$ is the volume density of electric charge.

We substitute the formula (1.29) into the formula (1.28) and obtain:

$$\begin{aligned} 0 &= -\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial t} \text{div}\vec{D}; \\ \frac{\partial}{\partial t} (\text{div}\vec{D} - \rho) &= 0 \end{aligned} \quad (1.30)$$

From the formula (1.30) we find:

$$\text{div}\vec{D} - \rho = \text{const} \quad (1.31)$$

The value of the constant in (1.31) is found from the initial conditions, which can always be chosen so that it is zero. Then we get:

$$\begin{aligned} \operatorname{div} \vec{D} - \rho &= \text{const}; \\ \operatorname{div} \vec{D} &= \rho \end{aligned} \quad (1.32)$$

Formula (1.32) is the fourth Maxwell equation. The presented conclusion is formal and reflects the system character of the group of equations. Therefore, consider the second derivation of the fourth Maxwell equation, which was originally known as the Ostrogradsky-Gauss theorem.

The Ostrogradsky-Gauss theorem can be formulated for the vector of the electrostatic field strength \vec{E} and for the vector of electric induction \vec{D} . In the system of SI units, the relationship between these vectors is expressed by the formula:

$$\begin{aligned} \vec{D} &= \varepsilon_0 \cdot \varepsilon \cdot \vec{E}; \\ D_n &= \varepsilon_0 \cdot \varepsilon \cdot E_n \end{aligned} \quad (1.33)$$

Here ε_0 - the electrostatic constant of vacuum (it determines the speed of propagation of electromagnetic waves in a vacuum); ε - the dielectric constant of the medium (substance) in which the electrostatic field is studied; D_n , E_n - the projections of the vector of electric induction and tension on the normal.

Using vector analysis, we introduce the concept of the flow of lines of the vector of electric induction:

$$N = \oint_S \vec{D} \cdot d\vec{S} = \oint_S D_n \cdot dS \quad (1.34)$$

As the integration surface, we select a sphere near the center, which has the electric charges that create the electric field. The radius of the sphere can be chosen so that the area occupied by charges is smaller than the entire sphere. Then the area occupied by the charges can be considered infinitesimal, and in this case we can assume that the charges are in the center of this sphere.

In this case, the radius vector of some point of the chosen sphere and the normal to the sphere at this point coincide in direction, then in formula (2.34) one can go from the index \vec{n} to the index of the radius vector \vec{r} . Then the flow of the lines of the vector of electric induction will be equal to:

$$N = \oint_S D_r \cdot dS \quad (1.35)$$

To find the vector of electrical induction, we use the formula of the electric field strength created by a point positive electric charge:

$$\vec{E} = \frac{q}{4\pi\varepsilon_0 \cdot \varepsilon} \cdot \frac{\vec{r}}{r^3};$$

$$E_r = \frac{q}{4\pi\varepsilon_0 \cdot \varepsilon} \cdot \frac{1}{r^2}$$
(1.36)

Now the flow of lines of the vector of electrostatic induction is determined by the formula:

$$N = \oint_S D_r dS = \oint_S \varepsilon_0 \varepsilon E_r dS = \oint_S \varepsilon_0 \varepsilon \frac{q}{4\pi\varepsilon_0 \varepsilon \cdot r^2} dS = \frac{q}{4\pi} \oint_S \frac{dS}{r^2}$$
(1.37)

In (1.37), the integral is taken over a closed surface inside which there are charges whose size is equal to q . They occupy a small area, which allows us to consider them as point charges.

The integral in (1.37) is the total solid angle, under which the inner surface of the sphere from its center is visible. Then we can write:

$$\oint_S \frac{dS}{r^2} = \begin{cases} 4\pi \\ 0 \end{cases}$$
(1.38)

In this case, the first value of the solid angle is obtained if the observation point is inside the sphere. Then 4π - this is the total solid angle, under which the inner surface of the sphere from its center is visible. In principle, this applies not only to the spherical surface, but also to the surface of any other shape, provided it is closed.

To explain the second case, it must be taken into account that the solid angle under which a certain surface is observed is considered positive if the direction of the view of the surface coincides with the direction of the normal to this surface. Conversely, the solid angle is considered negative if the direction of the view is opposite to the direction of the normal to the surface.

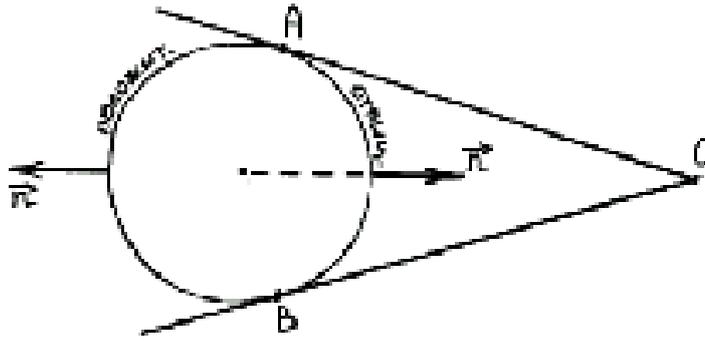


Fig.4. Definition of solid angle

Figure 2 shows a situation in which the charge creating the field is at a point, and from this point O a solid angle is defined, under which a closed surface is visible. It is seen that the left side of the surface is observed under a positive solid angle, and the right side of the surface is under a negative solid angle. The modules of these solid angles are the same. Then the solid angle, under which the entire surface from the point O is visible, is zero, which corresponds to the second case in formula (1.38).

We substitute the formula (1.38) in the formula (1.37) and get the fourth Maxwell equation in integral form:

$$N = \frac{q}{4\pi} \oint_S \frac{dS}{r^2} = \frac{q}{4\pi} \cdot \begin{cases} 4\pi \\ 0 \end{cases} = \begin{cases} q \\ 0 \end{cases};$$

$$N = \oint_S \vec{D} \cdot d\vec{S} = \begin{cases} q \\ 0 \end{cases}$$
(1.39)

The result can be interpreted as follows:

1. If there are electric charges inside a certain volume, the total flux of the vector of electric induction is equal to the value of the charge inside the volume. In the particular case, if there is no inside charge, then the total flux of the vector of electric induction is zero.
2. Lines of the vector of electrical induction start on one charge and end on other charges. It is conventionally accepted that the lines begin on positive charges, and end on negative charges.
3. If there are no electric charges inside a certain volume, then the number of lines entering the volume is equal to the number of lines emerging from the volume.

The fourth Maxwell equation can be written in a differential form. For this we use the vector analysis theorem (Gauss theorem):

$$\oint_S \vec{D} \cdot d\vec{S} = \int_V \text{div} \vec{D} \cdot dV \quad (1.40)$$

In addition, the charge can also be expressed in terms of the volumetric charge density:

$$q = \int_V \rho \cdot dV \quad (1.41)$$

In formulas (2.40) and (2.41), the volume V is equal to the volume enclosed inside the closed surface.

We substitute the expressions (2.40) and (2.41) into the formula (2.39):

$$N = \oint_S \vec{D} \cdot d\vec{S} = \int_V \text{div} \vec{D} \cdot dV = \int_V \rho \cdot dV \quad (1.42)$$

From the expression (1.42) we obtain:

$$\text{div} \vec{D} = \rho \quad (1.43)$$

In formulas (1.42) and (1.43), the case when the flux of the vector of electric induction was considered as a special case, corresponding to the fact that the charge inside the closed surface is zero.

The fifth and sixth Maxwell equations

Suppose we have a homogeneous and isotropic medium, for which dielectric permittivity ε and magnetic permeability μ are known. These quantities are related to the structure and behavior of atoms and molecules of the medium. For a vacuum, these quantities are equal to unity.

The fifth Maxwell equation establishes a connection between the electric field strength vector and the electric induction vector. It has the form:

$$\vec{D} = \varepsilon_0 \cdot \varepsilon \cdot \vec{E} \quad (1.44)$$

This formula includes a quantity called the electrostatic constant of the vacuum. This is a fundamental constant that determines the propagation velocity of electromagnetic waves in a vacuum.

$$\oint_S \vec{j} \cdot d\vec{S} = \int_V \text{div} \vec{j} \cdot dV \quad (1.45)$$

We substitute the formula (1.45) into the formula (1.43) and obtain:

$$\int_V \operatorname{div} \vec{j} \cdot dV = -\frac{\partial}{\partial t} \int_V \rho \cdot dV \quad (1.46)$$

Change the order of differentiation and integration on the right-hand side of (1.46) and get:

$$\int_V \left(\operatorname{div} \vec{j} + \frac{\partial \rho}{\partial t} \right) dV = 0 \quad (1.46)$$

The integral is zero for a nonzero integration volume if the integrand is zero:

$$\begin{aligned} \operatorname{div} \vec{j} + \frac{\partial \rho}{\partial t} &= 0; \\ \operatorname{div} \vec{j} &= -\frac{\partial \rho}{\partial t} \end{aligned} \quad (1.47)$$

Formula (1.47) is the law of conservation of charge in a differential form. Thus, the seventh Maxwell equation has the form:

1) in the integral form: $I = -\frac{\partial q}{\partial t}$

2) in the differential form: $\operatorname{div} \vec{j} = -\frac{\partial \rho}{\partial t}$

Both forms of recording express the law of conservation of electric charge.

The eighth Maxwell equation

The eighth Maxwell equation is a generalization of experimental data on the study of electric current. In integral form, this equation is Ohm's law for the complete chain and is written as:

$$I = \frac{E}{R + r} \quad (1.48)$$

Here E - the electromotive force of the current source, R - the resistance of the external part of the circuit, r - the internal resistance of the circuit or the resistance of the current source.

In order to get the eighth Maxwell equation in a differential form, let us consider in more detail the concept of an electric current. An electric current is any

directional motion of electric charges. This current is often called the conduction current.

Considering the role of environment in the study of electromagnetic phenomena, Faraday introduced the concept of the vector of electrical induction, which was originally called the vector of electrostatic displacement.

When Maxwell gave the ideas of Faraday a mathematical form, he introduced the notion of bias current. The bias current together with the conduction current form the total current in the circuit.

To explain the concept of the bias current, let us consider a capacitor in an electrical circuit. If the electrical circuit is a DC circuit, the capacitor is a break in the circuit. However, if the electrical circuit is an AC circuit, then a current flows through the capacitor, which is the bias current. The bias current is connected with the polarization of the dielectric inside the capacitor, or it is connected with an alternating electric field.

In order to get an expression for the bias current, we use the seventh Maxwell equation and write the current in the integral form:

$$I = -\frac{\partial q}{\partial t} \quad (1.49)$$

We express the current through the current density:

$$I = \oint \vec{j} \cdot d\vec{S} \quad (1.50)$$

We write the charge on the right-hand side of (1.50) using the Gauss theorem:

$$q = \oint_S \vec{D} \cdot d\vec{S} \quad (1.51)$$

We substitute the formulas (1.50) and (1.51) into the formula (1.49):

$$\oint_S \vec{j} \cdot d\vec{S} = -\frac{\partial}{\partial t} \oint_S \vec{D} \cdot d\vec{S} \quad (1.52)$$

On the right-hand side of (1.52) we interchange the operations of differentiation and integration, then we get:

$$\oint_S \vec{j} \cdot d\vec{S} = -\oint_S \frac{\partial}{\partial t} \vec{D} \cdot d\vec{S} \quad (1.53)$$

The formula (1.53) is transformed into the expression:

$$\oint_S \left(\vec{j} + \frac{\partial \vec{D}}{\partial t} \right) \cdot d\vec{S} = 0 \quad (1.54)$$

It follows from (1.64) that the quantity $\frac{\partial \vec{D}}{\partial t}$ is the density of a current, which is called the displacement current:

$$j_{cm} = \frac{\partial \vec{D}}{\partial t} \quad (1.55)$$

Then the formula (1.54), representing the law of conservation of charge, will have the form:

$$\oint_S \left(\vec{j} + j_{cm} \right) \cdot d\vec{S} = 0 \quad (1.56)$$

It follows from (1.56) that the total current density is equal to the sum of the conductivity current density and the bias current:

$$\vec{j} = \vec{j}_{np} + j_{cm} \quad (1.57)$$

Using the notion of current density and classical concepts of electrical conductivity, one can obtain the eighth Maxwell equation in a differential form:

$$\vec{j} = \sigma \cdot \left(\vec{E} + \vec{E}_{cmop} \right) \quad (1.58)$$

Here σ is the specific conductivity, which is related to the resistivity of conductors ρ by the formula

$$\sigma = \frac{1}{\rho} \quad (1.59)$$

\vec{E} - the intensity of the electric field created by electric charges is the intensity of the field created by external forces due to chemical, \vec{E}_{cmop} - thermal and other processes.

1.3 Equations of electrodynamics

As the basic mathematical model, the "two-dimensional slice", most widely used in the study of georadar, was chosen in which the horizontal component

$$E_z = E(x, y, t) \quad (1.2.1)$$

the electric field is described by the wave equation

$$\mu\varepsilon \frac{\partial^2 E_z}{\partial t^2} + \mu\sigma \frac{\partial E_z}{\partial t} = \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} - g(x, y)V'(t), \quad x < 0, t > 0 \quad (1.2.2)$$

$$E_z|_{t < 0} = 0 \quad (1.2.3)$$

Here: ε is the dielectric constant of the medium.

In the direct problem, it is required to construct $E(x, y, t)$, knowing the function ε and the function $g(x, y)$, $V'(t)$, describing the source of the electromagnetic oscillations emitted by the transmitting antenna.

In the inverse problem, it is required to determine the function of additional information

$$E(x, 0, t) = f(y, t) \quad (1.2.4)$$

which is recorded by the receiving antenna.

Basic mathematical and physical ideas of work:

- 1) Construction of a mathematical model and a comparative analysis taking into account the data of the Loza-G georadar.
- 2) The idea of continuing the electromagnetic field toward heterogeneities. One of the main obstacles for the localization of underground objects is the upper part of the soil, which lies above the objects sought. Passing through this area, electromagnetic waves reflected from various objects interact with each other, can amplify or, conversely, mutually decrease. One way to overcome this problem is to continue solving the system of Maxwell's equations from the earth's surface in the direction of the location of the objects sought. The continuation problem is one of the most complicated and ill-posed problems in mathematical physics, complicated in this case by the presence of an electromagnetic field decay in conducting media. In this paper, we use the idea of reducing the extension problem to a linear inverse problem with respect to some direct boundary value problem. The results of calculations of the solution of the continuation problem showed that the obtained data coincide with the

real data of the Loza-G georadar. Moreover, the results of solving the field continuation problem help in the interpretation of radargrams.

3) The inverse problem is reduced to the operator equation $Aq = f$, whose solution is sought by minimizing the corresponding objective functional $J(q) = \langle Aq-f, Aq-f \rangle$.

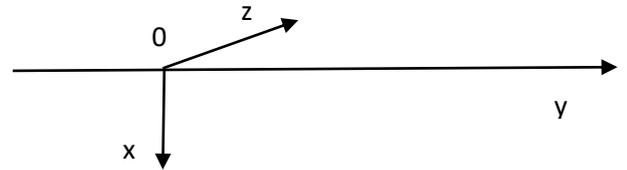
4) Minimization of the target function is sought on the basis of gradient methods using the conjugate problem, the finite element method, and the finite difference method.

In this section, we consider a mathematical model for the operation of a subnanosecond probe placed in an axisymmetric well. The Maxwell equations are considered in the Cartesian coordinate system.

Consider a geophysical model of the medium, consisting of two half-spaces:

$$R_-^3 = \{x, y, z \in R^3, \quad x < 0\} - \text{air};$$

$$R_+^3 = \{x, y, z \in R^3, \quad x > 0\} - \text{land}.$$



Consider the system of Maxwell equations

$$\begin{cases} \varepsilon \frac{\partial E}{\partial t} - \text{rot } H + \sigma E + j^{cm} = 0, \\ \mu \frac{\partial H}{\partial t} + \text{rot } E = 0, \end{cases} \quad (x, y, z) \in R^3, \quad x \neq 0, \quad t > 0. \quad (1.2.5)$$

Here the positive functions $\varepsilon(x, y, z)$, $\sigma(x, y, z)$ and $\mu(x, y, z)$ the permittivity, conductivity and magnetic permeability of the medium, respectively. We believe that there are no electromagnetic oscillations up to the time $t = 0$ point:

$$(E, H)|_{t < 0} \equiv 0, \quad j^{cm}|_{t < 0} \equiv 0 \quad (1.2.6)$$

and then induced by an external current $j^{cm}(x, y, z, t)$

Let's consider one of the simplest versions of the problem, when ε , σ and μ , depend only on the depth x and one horizontal variable y , and the source of the external current is a rather long (infinite) cable centered and stretched along the axis z :

$$(E, H)|_{t < 0} \equiv 0, \quad j^{cm}(x, y, z, t) = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} g(x, y) V(t). \quad j^{cm}|_{t < 0} \equiv 0 \quad (1.2.7)$$

Here the function $g(x, y)$ describes the transverse dimensions of the source.

In this case, neglecting the effect of the cable ends, we conclude that in the system of Maxwell's equations only three components remain nonzero E_z, H_x, H_y and the system will have the form [24]:

$$\begin{aligned} \varepsilon \frac{\partial E_z}{\partial t} + \frac{\partial H_x}{\partial y} - \frac{\partial H_y}{\partial x} + \sigma E_z + g(x, y)V(t) &= 0, \\ \mu \frac{\partial H_x}{\partial t} + \frac{\partial E_z}{\partial y} &= 0, \\ \mu \frac{\partial H_y}{\partial t} - \frac{\partial E_z}{\partial x} &= 0. \end{aligned} \quad (1.2.8)$$

After excluding the partial derivatives of the components H_x and H_y from the first equation, we obtain E_z with respect to the second-order equation

$$\begin{aligned} \mu \varepsilon \frac{\partial^2 E_z}{\partial t^2} + \mu \sigma \frac{\partial E_z}{\partial t} &= \frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} - g(x, y)V'(t), \\ (x, y, z) \in R^3, \quad x \neq 0, \quad t > 0. \end{aligned} \quad (1.2.9)$$

The main physical example is the two-dimensional electrodynamics problem considered above. In equation (1.2.3) we denote $v(x, y, \tau) = E_z(x, y, t)$, $\mu = \text{const}$ and obtain the following equation:

$$\varepsilon v_{\tau\tau} + \sigma v_{\tau} = \frac{1}{\mu} (v_{xx} + v_{yy}) \quad (1.2.10)$$

We introduce a new variable $t = \frac{\tau}{\sqrt{\varepsilon\mu}}$. For a function $u(x, y, t) = v(x, y, \tau)$ we get,

$$u_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}u_t = u_{xx} + u_{yy} \quad (1.2.11)$$

2 THE PROBLEM OF CONTINUATION OF THE ELECTROMAGNETIC FIELD

In this chapter, problems of continuation of wave fields from part of the boundary located on the surface of the Earth will be investigated.

The continuation problems relate to ill-posed problems of mathematical physics. The foundations of the theory of inverse and ill-posed problems were laid down in the works of AN Tikhonov, MM Lavrentev, VK Ivanov, as well as their students and followers. In many inverse and ill-posed problems, the unknown inhomogeneities are located at some depth below the medium layer, the parameters of which are known (in geophysics these are usually homogeneous or horizontally layered media). In this case, an important tool for practitioners is the task of continuing geophysical fields from the earth's surface to the occurrence of inhomogeneities.

2.1 Formulation of the problem

We consider the problem of extending the electromagnetic field from a part of the boundary $x=0$ in the region, where $\Omega = \{(x, y) : x \in (0, h), y \in (0, L)\}$: $(x, y) \in \Omega, t \in (0, T)$;

$$u_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}u_t = u_{xx} + u_{yy} \quad (x, y) \in \Omega, \quad t \in (0, T), \quad (1)$$

$$u|_{t=0} = 0, \quad u_t|_{t=0} = 0, \quad (x, y) \in \Omega \quad (2)$$

$$u_x(0, y, t) = g(y, t), \quad y \in (0, L), \quad t \in (0, T), \quad (3)$$

$$u(0, y, t) = f(y, t), \quad y \in (0, L), \quad t \in (0, T), \quad (4)$$

$$u(x, 0, t) = u(x, L, t) = 0, \quad y \in (0, L), \quad t \in (0, T), \quad (5)$$

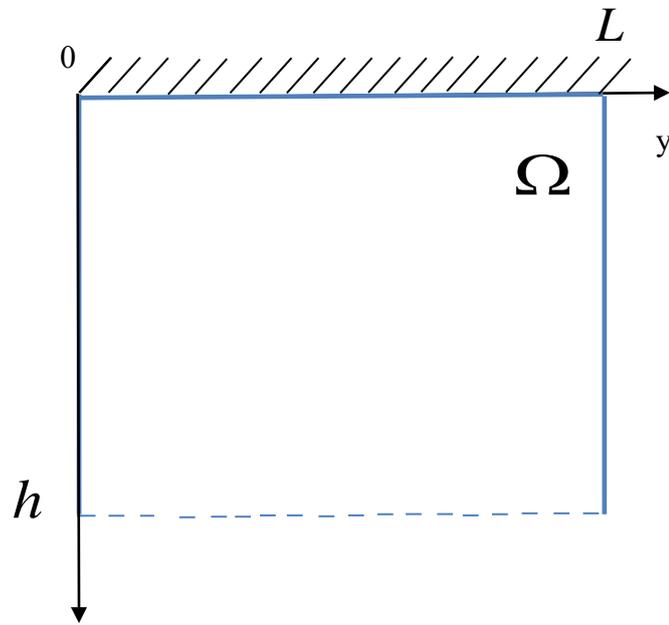


Figure 5 - Area Ω

The problem (1) - (5) is incorrect. The problem consists in finding the function $u(x, y, t)$ from a part of the boundary $x=0$ in the domain $\Omega \times (0, T)$;

Physical statement of the problem: a source of electromagnetic waves is switched on at the boundary $x=0$ of the environment of the investigated region Ω (3). It is believed that the intensity of electromagnetic waves on the surface $x=0$ in the course of time $t \in (0, T)$. It is known that this is the condition (4). We assume that until the time $t = 0$ the medium was at rest. Then condition (2) is satisfied.

$u(x, y, t)$ - horizontal component of the electric field strength vector of the electromagnetic field.

The basic idea of solving the continuation problem is to reduce to the inverse problem with respect to the ill-posed problem for the initial equation and to minimize the target functional.

2.2 The direct and inverse problem

In addition, we specify the boundary conditions:

$$u(h, y, t) = q(y, t), y \in (0, L), \quad t \in (0, T) \quad (5)$$

$$u(x, 0, t) = u(x, L, t) = 0, x \in (0, h), \quad t \in (0, T) \quad (6)$$

The direct problem is included in the definition of the function $u(x, y, t)$ from equation (1) with the initial-boundary conditions (2), (3), (5), (6).

The inverse problem is included in the definition of the function $q(x, y)$ from the relations (1) - (2), (4) and on additional information about the solution of the direct problem:

$$u(0, y, t) = f(y, t), y \in (0, L), \quad t \in (0, T) \quad (7)$$

Returning to the direct problem (1) - (3), (5), (6) from equation (1) and condition (2), (3), (5), (6), (7).

2.3 Target functional and its gradient

The solution of the continuation problem for the equation of electrodynamics by the Landver method

We introduce the operator A as follows

$$\begin{aligned} A: q(x, y) &\rightarrow f(y, t) \\ A: W_2^1(\Omega) &\rightarrow W_2^1((0, L) \times (0, T)). \end{aligned}$$

When the inverse problem (1) - (3), (5) - (7) is written in the operator form

$$Aq = f. \quad (8)$$

We introduce the objective functional

$$J(q) = \|Aq - f\|_{L_2((0, L) \times (0, T))}^2 = \int_0^L \int_0^T [u(0, y, t; q) - f(y, t)]^2 dy dt. \quad (9)$$

The minimization of the objective functional (9) is carried out by the Landver method:

$$q_{n+1} = q_n - \alpha J' q_n, \quad (10)$$

where $\alpha \in \left(0, \frac{1}{\|A\|^2}\right)$ - startup parameter [25].

Calculation of the gradient of the target functional

Theorem 1. Let the coefficients of equation (1) and the function be continuous functions, then the gradient of the functional (10) is the main part of the increment of the functional $J'(q) = \langle J', \delta q \rangle_{L_2} + o(\|\delta q\|_{L_2}^2)$ and has the form [26]:

$$J'q_n = \psi_t(x, y, x) - \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}\psi(x, y, x) \quad (11)$$

where the function $\psi(x, y, t)$ is the solution of the auxiliary (conjugate) problem:

$$\psi_{tt} - \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}\psi_t = \psi_{xx} + \psi_{yy}, \quad (x, y, t) \in \Delta(L) \quad (12)$$

$$\psi(x, y, t) = 0, \quad x \in (0, L), y \in (0, L) \quad (13)$$

$$\psi_x(0, y, t) = 2[u(0, y, t) - f(y, t)], \quad y \in (0, L), t \in (0, L) \quad (14)$$

$$\psi_x(0, y, t) = 2[u(0, y, t) - f(y, t)], \quad y \in (0, L), t \in (0, L) \quad (15)$$

Substantiation. Set the increment $q_n + \delta q_n$, then

$$\delta u = \tilde{u} - u = u(x, y, t; q_n + \delta q_n) - u(x, y, t; q_n). \quad (16)$$

Using the notation (16), calculate the increment of the target functional $J(q)$ in (9).

$$\begin{aligned} J(q_n + \delta q_n) - J(q_n) &= \int_0^{L_y} \int_0^{2L_x} [u(0, y, t; q_n + \delta q_n) - f(y, t)]^2 dy dt - \\ &- \int_0^{L_y} \int_0^{2L_x} [u(0, y, t; q_n) - f(y, t)]^2 dy dt = \int_0^{L_y} \int_0^{2L_x} [u(0, y, t; q_n + \delta q_n) - u(0, y, t; q_n)] \times \\ &\times [u(0, y, t; q_n + \delta q_n) - f(y, t) + u(0, y, t; q_n) - f(y, t)] dy = \\ &= \int_0^{L_y} \int_0^{2L_x} \delta u(0, y, t; q_n) 2[u(0, y, t; q_n) - f(y, t)] dy + o(\|\delta u\|). \end{aligned} \quad (17)$$

Consider the formulation of the perturbed problem for equations (1) - (5).

$$\tilde{u}_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}\tilde{u}_t = \tilde{u}_{xx} + \tilde{u}_{yy} \quad (18)$$

$$\tilde{u}_x(0, y, t) = g(y, t), \quad (19)$$

$$\tilde{u}(x, y, x) = q_n + \delta q_n, \quad (20)$$

$$\tilde{u}(x, 0, t) = u(x, L_y, t) = 0. \quad (21)$$

From the relations (18) - (21), we subtract relations (1) - (5) and, taking (16) into account; we obtain the following problem for the increment δu :

$$\delta u_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}\delta u_t = \delta u_{xx} + \delta u_{yy} \quad (22)$$

$$\delta u_x(0, y, t) = 0, \quad (23)$$

$$\delta u(x, y, x) = \delta q_n, \quad (24)$$

$$\delta u(x, 0, t) = u(x, L_y, t) = 0. \quad (25)$$

We multiply (21) by an arbitrary function $\psi(x, y, t)$ and integrate over the domain Ω .

$$\begin{aligned} 0 = & \iiint_{\Omega} (\delta u_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}\delta u_t - \delta u_{xx} - \delta u_{yy}) \psi dx dy dt = \int_0^{L_y} \int_0^{L_x} \int_x^{2L_x-x} \psi \delta u_{tt} dt dx dy \\ & - \int_0^{L_y} \int_0^{L_x} \int_0^t \psi \delta u_{xx} dx dt dy - \int_0^{L_y} \int_{L_x}^{2L_x} \int_0^{2L_x-t} \psi \delta u_{xx} dx dt dy - \int_0^{L_x} \int_x^{2L_x-x} \int_0^{L_y} \psi \delta u_{yy} dy dt dx \\ & + \int_0^{L_y} \int_0^{L_x} \int_x^{2L_x-x} \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}} \psi \delta u_t dt dx dy \end{aligned}$$

We integrate this expression by parts, we obtain:

$$\begin{aligned}
& \int_0^{L_y} \int_0^{L_x} [(\psi \delta u_t)(x, y, 2L_x - x) - \underline{(\psi \delta u_t)(x, y, x)} - (\psi_t \delta u)(x, y, 2L_x - x) + \\
& + \underline{(\psi_t \delta u)(x, y, x)} + \int_x^{2L_x - x} \psi_{tt} \delta u dt] dx dy + \int_0^{L_y} \int_0^{L_x} \frac{\sigma \sqrt{\mu}}{\sqrt{\varepsilon}} [(\psi \delta u_t)(x, y, 2L_x - x) - \\
& - \underline{(\psi \delta u_t)(x, y, x)} - \int_x^{2L_x - x} \psi_t \delta u dt] dx dy - \int_0^{L_y} \int_0^{L_x} \frac{\sigma \sqrt{\mu}}{\sqrt{\varepsilon}} [\underline{(\psi \delta u_x)(t, y, t)} - \\
& - (\psi \delta u_x^{\square})(0, y, t) - \underline{(\psi_x \delta u)(t, y, t)} + (\psi_x \delta u)(0, y, t) + \int_0^t \psi_{xx} \delta u dx] dt dy - \\
& - \int_0^{L_y} \int_{L_x}^{2L_x} \frac{\sigma \sqrt{\mu}}{\sqrt{\varepsilon}} [(\psi \delta u_x)(2L_x - t, y, t) - (\psi \delta u_x^{\square})(0, y, t) - (\psi_x \delta u)(2L_x - t, y, t) + \\
& + (\psi_x \delta u)(0, y, t) + \int_0^{2L_x - t} \psi_{xx} \delta u dx] dt dy - \int_0^{L_x} \int_x^{2L_x - x} [(\psi \delta u_y)(x, L_y, t) - (\psi \delta u_y)(x, 0, t) - \\
& - (\psi_y \delta u^{\square})(x, L_y, t) + (\psi_y \delta u^{\square})(x, 0, t) + \int_0^{L_y} \psi_{yy} \delta u dy] dt dx
\end{aligned}$$

Taking into account (24) and (25) and by virtue of the fact that

$$\delta u_x(x, y, x) + \delta u_t(x, y, x) = \frac{d\delta u}{dx} \Big|_{\frac{dt}{dx}=1} = (\delta q)_x \text{ (directional derivative } t=x), \text{ integrating}$$

by parts, we obtain:

$$\begin{aligned}
0 = & \iiint_{\Omega} \left(\psi_{tt} - \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}} \psi_t - \psi_{xx} - \psi_{yy} \right) \delta u dx dy dt + \\
& + \int_0^{L_y} \int_0^{L_x} [(\psi \delta u_t)(x, y, 2L_x - x) - (\psi_t \delta u)(x, y, 2L_x - x) + \\
& + (\psi_t \delta u)(x, y, x)] dx dy + \int_0^{L_y} \int_0^{2L_x} (\psi_x \delta u)(0, y, t) dt dy + \\
& + \int_0^{L_y} \int_0^{L_x} \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}} [(\psi \delta u_t)(x, y, 2L_x - x) - \underline{(\psi \delta u_t)(x, y, x)}] dx dy - \\
& - \int_0^{L_y} \int_0^{L_x} (\psi_x \delta u)(t, y, t) dt dy - \int_0^{L_y} \int_{L_x}^{2L_x} [(\psi \delta u_x)(2L_x - t, y, t) - \\
& - (\psi_x \delta u)(2L_x - t, y, t)] dt dy - \int_0^{L_y} (\psi \delta u) \Big|_0^{L_x} dy + \int_0^{L_y} \int_0^{L_x} (\psi_x \delta u)(t, y, t) dx dy - \\
& - \int_0^{L_x} \int_x^{2L_x - x} [(\psi \delta u_y)(x, L_y, t) - (\psi \delta u_y)(x, 0, t)] dt dx
\end{aligned}$$

From this follows the formulation of the adjoint problem

$$\psi_{tt} - \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}} \psi_t = \psi_{xx} + \psi_{yy}, \quad 26$$

$$\psi(x, y, t) = 0, \quad 27$$

$$\psi_x(0, y, t) = 2[u(0, y, t) - f(y, t)], \quad 28$$

$$\psi_x(0, y, t) = 2[u(0, y, t) - f(y, t)], \quad 29$$

Then, taking into account (17), we obtain

$$\langle \delta q_n, J'q_n \rangle = \int_0^{L_y} \int_0^{L_x} (\psi_t(x, y, t)) \delta q dx dy.$$

By definition [28, p.260], the main part of the increment of the functional is a gradient, i.e.

$$J'q_n = \psi_t(x, y, t).$$

Here $\psi(x, y, t)$ there is a solution of the dual problem (12)-(15).
The theorem is proved.

2.4 Computational algorithm for solving the inverse problem

- 1 Choose the initial approximation q_0 .
- 2 Assuming that we know q_n , we solve the direct problem

$$u_{tt} + \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}u_t = u_{xx} + u_{yy}, \quad (x, y) \in \Omega, \quad t \in (0, T), \quad (1)$$

$$u|_{t=0} = 0, \quad u_t|_{t=0} = 0, \quad (x, y) \in \Omega \quad (2)$$

$$u_x(0, y, t) = g(y, t), \quad y \in (0, L), \quad t \in (0, T), \quad (3)$$

$$u(0, y, t) = f(y, t), \quad y \in (0, L), \quad t \in (0, T), \quad (4)$$

$$u(x, 0, t) = u(x, L, t) = 0, \quad y \in (0, L), \quad t \in (0, T), \quad (5)$$

- 3 We calculate the value of the functional:

$$J(q) = \|Aq - f\|_{L_2((0,L) \times (0,T))}^2 = \int_0^L \int_0^T [u(0, y, t; q) - f(y, t)]^2 dy dt.$$

- 4 If the value of the target functional is not enough, then we solve the conjugate problem:

$$\psi_{tt} - \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}\psi_t = \psi_{xx} + \psi_{yy}, \quad (26)$$

$$\psi(x, y, t) = 0, \quad (27)$$

$$\psi_x(0, y, t) = 2[u(0, y, t) - f(y, t)], \quad (28)$$

$$\psi_x(0, y, t) = 2[u(0, y, t) - f(y, t)], \quad (29)$$

5 Calculate the gradient of the functional:

$$J'(q) = \psi_x(x, y, t)$$

6 Calculate the next approximation

$$q_{n+1} = q_n - \alpha J'q_n$$

and go to point 2.

3 NUMERICAL SOLUTION OF THE PROBLEM OF CONTINUATION OF THE ELECTROMAGNETIC FIELD

3.1 The Fourier method and the method of finite differences for solving the direct problem and conjugate problem

We denote by $\sigma_1 = \frac{\sigma\sqrt{\mu}}{\sqrt{\varepsilon}}$, then equation (1) can be written in the following form:

$$u_{tt} + \sigma_1 u_t = u_{xx} + u_{yy}.$$

We expand the function $u(x, y, t)$ и $\sigma_1(x, y)$ in a Fourier series in the variable y and we get:

$$u(x, y, t) = \sum u^k(x, t)e^{iky}$$

$$\sigma_1(x, y) = \sum \sigma_1^k(x)e^{iky}$$

Then:

$$\sum u_{tt}^k e^{iky} + \sum_{n,k} \sigma_1^n e^{iny} u_t^k e^{iky} = \sum u_{xx}^k e^{iky} - \sum_k k^2 u^k(x, t) e^{iky}.$$

We introduce the following notation:

$$n + k = k', \quad n = k' - k$$

$$u_{tt}^k + \sum_{k'} \sigma_1^{k'-k} u_t^{k'} = u_{xx}^k - k^2 u^k$$

Taking into account that the function U has the form:

$$U = \begin{pmatrix} u_0 \\ u_1 \\ \dots \\ u_N \end{pmatrix} \quad n = 10, 15, 20.$$

We obtain the following problem:

$$U_{tt} - \Sigma(x)U_t = U_{xx} + E_N U, \quad (3.1)$$

$$U_x|_{x=0} = G(t), \quad (3.2)$$

$$U|_{t=x} = Q(x), \quad (3.3)$$

where

$$E_N = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1^2 & 0 & 0 & 0 \\ 0 & 0 & 2^2 & 0 & 0 \\ 0 & 0 & 0 & \dots & 0 \\ 0 & 0 & 0 & 0 & N^2 \end{pmatrix}, \quad \Sigma(x) = \begin{pmatrix} \sigma_1^N & \sigma_1^{N-1} & \sigma_1^{N-2} & \dots & \sigma_1^0 \\ \sigma_1^{N-1} & \sigma_1^{N-2} & \sigma_1^{N-3} & \dots & \sigma_1^1 \\ \sigma_1^{N-2} & \sigma_1^{N-3} & \sigma_1^{N-4} & \dots & \sigma_1^2 \\ \dots & \dots & \dots & \dots & \dots \\ \sigma_1^0 & \sigma_1^1 & \sigma_1^2 & \dots & \sigma_1^N \end{pmatrix},$$

The direct problem (3.1) - (3.3) - is required to be determined from the given functions $G(t)$ and $Q(x)$.

The inverse problem is included in the definition of the function $Q(x)$ from the relations (3.1) - (3.3) for additional information on the solution of the direct problem:

$$U|_{x=0} = F(t), \quad (3.4)$$

We replace the derivatives in Eq. (3.1) by finite-difference analogues and obtain:

$$\frac{U_i^{k+1} - 2U_i^k + U_i^{k-1}}{h^2} + \Sigma_i \frac{U_i^{k+1} - U_i^{k-1}}{2h} = \frac{U_{i+1}^k - 2U_i^k + U_{i-1}^k}{h^2} - E_N U_i^k,$$

$$U_i^{k+1} + U_i^{k-1} + \frac{h}{2} \Sigma_i (U_i^{k+1} - U_i^{k-1}) = U_{i+1}^k + U_{i-1}^k - h^2 E_N U_i^k$$

$$U_i^k = \frac{U_i^{k+1} + U_i^{k-1}}{2}$$

Next we get:

$$\underline{U_i^{k+1}} + U_i^{k-1} + \frac{h}{2} \Sigma_i (\underline{U_i^{k+1}} - U_i^{k-1}) = U_{i+1}^k + U_{i-1}^k - h^2 E_N (\underline{U_i^{k+1}} - U_i^{k-1})$$

$$\left(I + \frac{h}{2} \Sigma_i + \frac{h}{2} E_N \right) U_i^{k+1} = U_{i+1}^k + U_{i-1}^k - \left(I - \frac{h}{2} \Sigma_i + \frac{h}{2} E_N \right) U_i^{k-1}$$

We introduce the notation

$$M_i = I + \frac{h}{2} \Sigma_i + \frac{h}{2} E_N \quad (3.5)$$

$$P_i = I - \frac{h}{2} \Sigma_i + \frac{h}{2} E_N, \quad (3.6)$$

and we get:

$$M_i U_i^{k+1} = U_{i+1}^k + U_{i-1}^k - P_i U_i^{k-1}$$

Then we get:

$$U_i^{k+1} = M_i^{-1} \left(U_{i+1}^k + U_{i-1}^k - P_i U_i^{k-1} \right).$$

We write the boundary condition (3.6)

$$U_x|_{x=0} = G(t),$$

$$\begin{aligned} U(h, kh) &= U_1^k = U_0^k + hU_x|_{x=0} + \frac{h^2}{2} U_{xx} = U_0^k + hG^k + \frac{h^2}{2} \left(U_{tt} + \Sigma(0)U_t + E_N U|_{x=0} \right) = \\ &= \underline{U_0^k} + hG^k + \frac{h^2}{2} \left(\frac{U_0^{k+1} - 2U_0^k + U_0^{k-1}}{h^2} + \Sigma_0 \frac{U_0^{k+1} - U_0^{k-1}}{2h} + E_N U_0^k \right) = \\ &hG^k + \frac{U_0^{k+1} - U_0^{k-1}}{2} + \frac{h}{4} \Sigma_0 (U_0^{k+1} - U_0^{k-1}) + \frac{h^2}{2} E_N U_0^k \end{aligned}$$

$$U_0^k = \frac{U_0^{k+1} + U_0^{k-1}}{2}$$

$$U_1^k = hG^k + \frac{U_0^{k+1} - U_0^{k-1}}{2} + \frac{h}{4}\Sigma_0(U_0^{k+1} - U_0^{k-1}) + \frac{h^2}{2}E_N\left(\frac{U_0^{k+1} + U_0^{k-1}}{2}\right)$$

$$U_1^k = hG^k + \frac{1}{2}\left(I + \frac{h}{2}\Sigma_0 + \frac{h^2}{2}E_N\right)U_0^{k+1} + \frac{1}{2}\left(I - \frac{h}{2}\Sigma_0 + \frac{h^2}{2}E_N\right)U_0^{k-1}$$

Taking into account the values of (3.5) and (3.6), we obtain:

$$U_1^k = hG^k + \frac{1}{2}M_0U_0^{k+1} + \frac{1}{2}P_0U_0^{k-1},$$

$$M_0U_0^{k+1} = 2U_1^k - 2hG^k - P_0U_0^{k-1}.$$

Next we get:

$$U_0^{k+1} = M_0^{-1}(2U_1^k - 2hG^k - P_0U_0^{k-1}),$$

where $U_i^i = Q_i$ - the solution of the direct problem.

3.2 Analysis of the numerical solution in the presence of localized objects

In this sub-section we will show that the continuation of the solution to the depth $x = h$ helps to localize the inhomogeneities below the line $x = h$.

Calculations were carried out in the area of $(x, y) = (0,10) \times (0,12)$ m, observation time 50 n/s. The time step is $h_t = 0.0047$ n/s. Step over the space variables $h_x = h_y = 0.01$ M.

Calculations are carried out for three different media: with one inhomogeneity, with two inhomogeneities and three inhomogeneities located at a depth of 6 m.

Example 1 A numerical experiment for the task of extending a field to a depth $x = h$ in a medium with one inhomogeneity (Figure 4)

Environment parameters $\varepsilon = 1$, $\sigma = 0.01$, values of parameters inside the inhomogeneity $\varepsilon = 20$, $\sigma = 0.001$. The inhomogeneity is at depth $x = 6$ M (Figure 6).

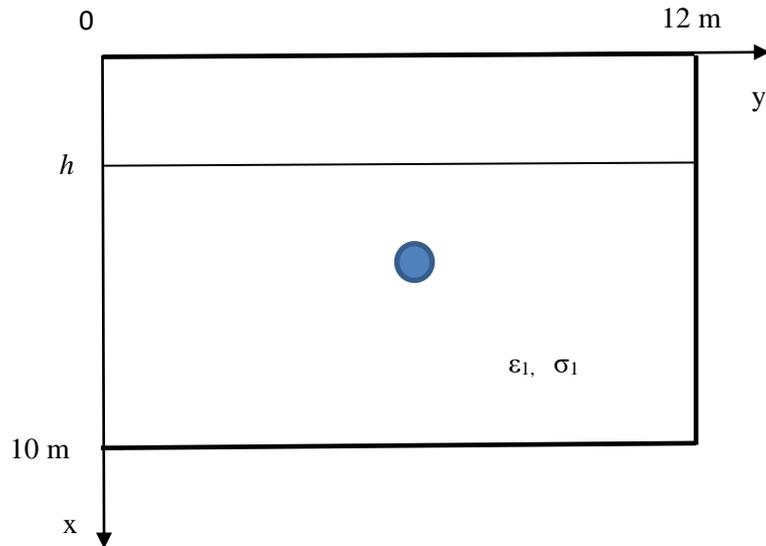


Figure 6 - Scheme of the experiment with one inhomogeneity

First we solve the direct problem (1) - (4) and obtain the additional condition (7) on the boundary $x = 0$. Then solve the reverse problem of extending the field to the depth $h = 3$ m, towards alleviating the inhomogeneity (1) - (2), (4) for additional information about the solution direct problem (7). The results of numerical calculations with one heterogeneity are shown in Figures 7-8.

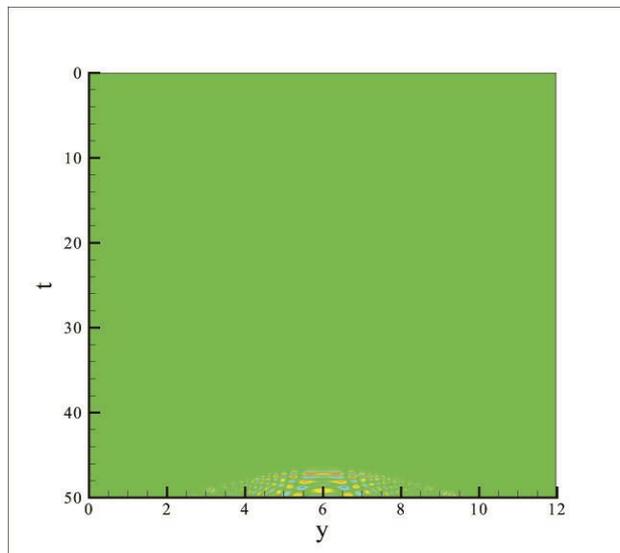


Figure 7 - Observed anomalous field at the boundary $x = 0$ m

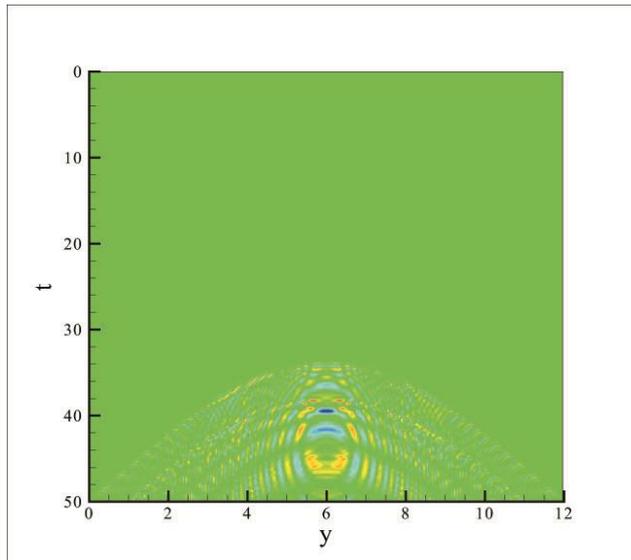


Figure 8 - Observed anomalous field at the boundary $x = 3$

Example 2 A numerical experiment for the task of extending a field to a depth $x = h$ in a medium with one inhomogeneity (Figure 7)

Environment parameters $\varepsilon = 1$, $\sigma = 0.01$, values of parameters inside the inhomogeneity $\varepsilon = 20$, $\sigma = 0.001$. The inhomogeneity is at depth $x = 6\mathcal{M}$ (Figure 9).

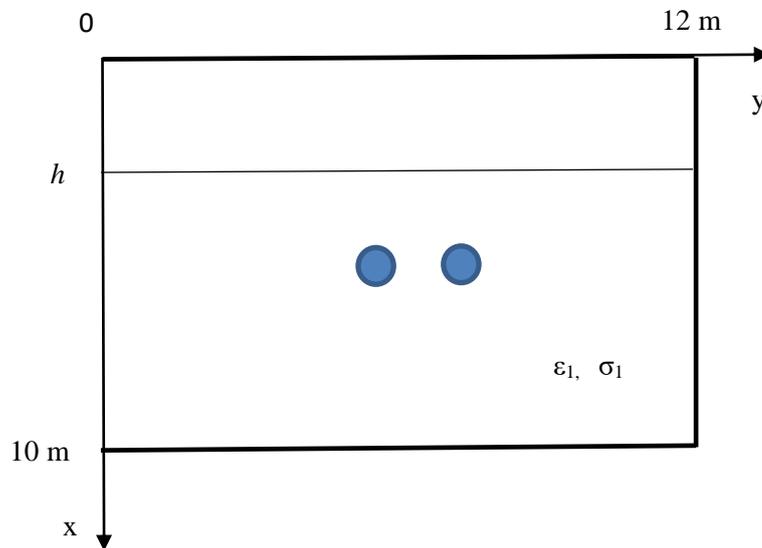


Figure 9 - Scheme of the experiment with two in homogeneities

First we solve the direct problem (1) - (4) and obtain the additional condition (7) on the boundary $x = 0$. Then solve the reverse problem of extending the field to the

depth $h = 3$ m, towards alleviating the inhomogeneity (1) - (2), (4) for additional information about the solution direct problem (7). The results of numerical calculations with one heterogeneity are shown in Figures 10-11.

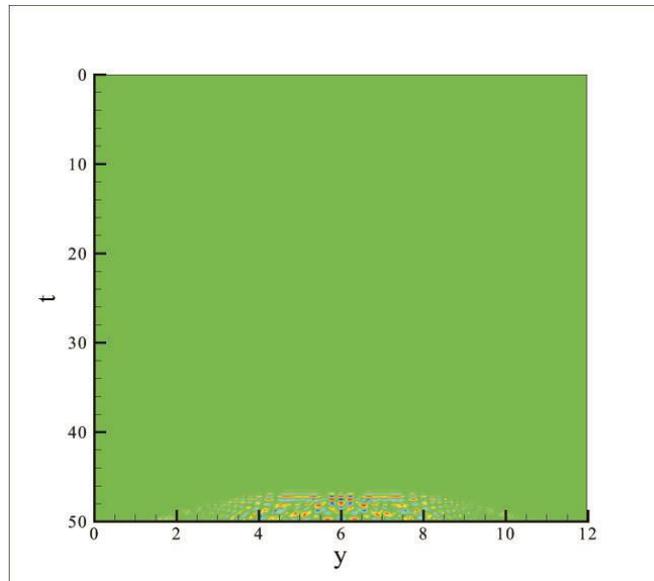


Figure 10 - Observed anomalous field at the boundary $x = 0$ m

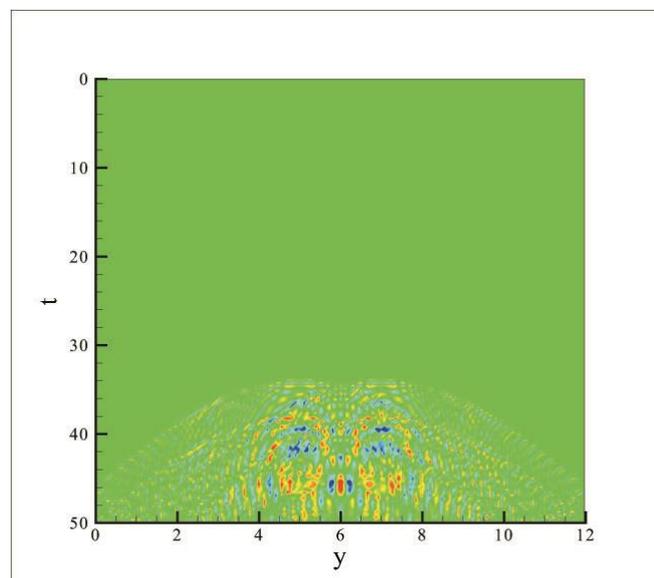


Figure 11 - Observed anomalous field at the boundary $x = 3$ m

The results of numerical calculations with two inhomogeneity show that the continuation of the field at a depth of m helps determine and distinguish two inhomogeneity at a depth of 6 m .

Example 3 A numerical experiment for the task of extending a field to a depth $x = h$ in a medium with three inhomogeneity (Figure 10)

Environment parameters $\varepsilon = 1$, $\sigma = 0.01$, values of parameters inside the inhomogeneity $\varepsilon = 20$, $\sigma = 0.001$. The inhomogeneity is at depth $x = 6\text{ m}$ (Figure 12).

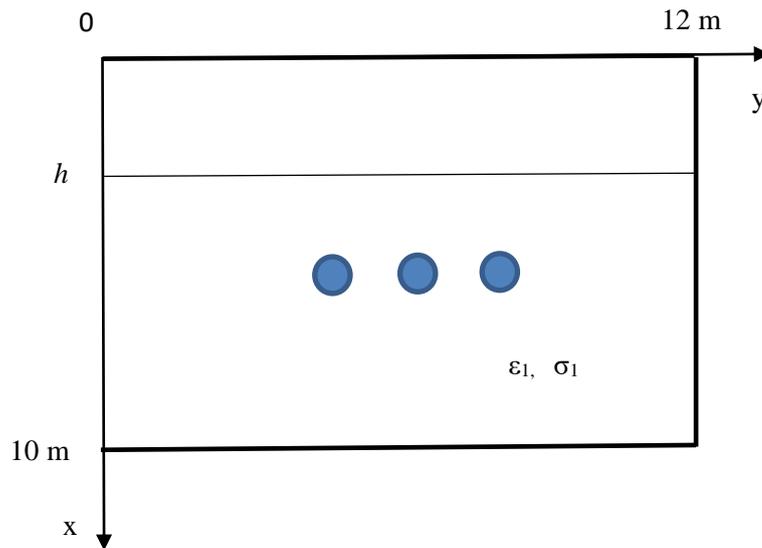


Figure 12 - Scheme of the experiment with three inhomogeneity

First we solve the direct problem (1) - (4) and obtain the additional condition (7) on the boundary $x = 0$. Then solve the reverse problem of extending the field to the depth $h = 3\text{ m}$, towards alleviating the inhomogeneity (1) - (2), (4) for additional information about the solution direct problem (7). The results of numerical calculations with one heterogeneity are shown in Figures 11-12.

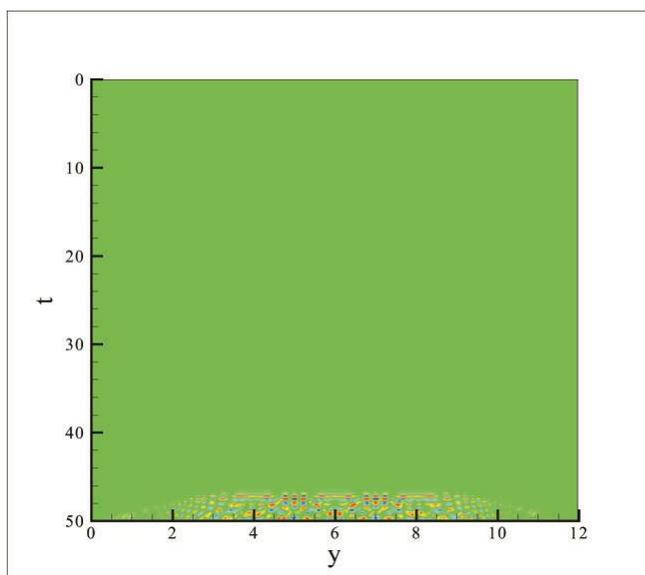


Figure 13 - Observed anomalous field at the boundary $x = 0$ m

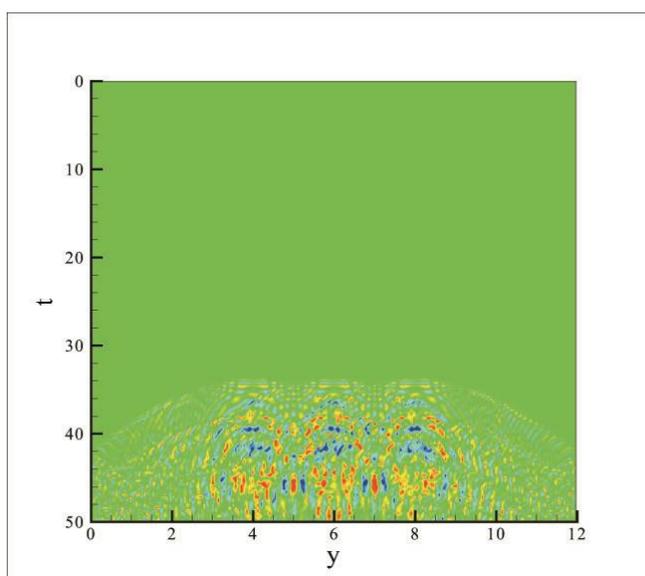


Figure 14 - Observed anomalous field at the boundary $x = 3$

The results of numerical calculations with two inhomogeneity show that the continuation of the field at a depth of m helps determine and distinguish two inhomogeneity at a depth of 6 m.

Example 4 An experiment carried out with the aid of a georadar

In the experiment carried out with the aid of a georadar, in order to obtain a similar setting of the examples 1-3, the following measurements were made by the complex-the Loza-V georadar. Two heterogeneities in the form of metal cylinders, with dimensions $(0.08) \times (0.3) \text{ m}^2$, buried to a depth of 0.55 m. The measurement length is $y = 6 \text{ m}$ and is divided into 20 points in 0.3 m steps (Figure 15).

Measurement procedure - first the source is at the 0th point, and the receiver measures at all other points. Then we place the source on the 1st point, and the receiver measures all other points. Thus, we pass all 20 points. The resulting radargrams are translated into a numerical format and summarized.

First, we measure the georadar and obtain a field in a homogeneous medium without inclusion (Figure 15). Then, let us measure the field of the medium with inhomogeneities by the georadar (Figure 2.15). Let us calculate the difference between the radarogram of a medium with two inhomogeneities and a radarogram of the medium without inclusion (Figure 15).

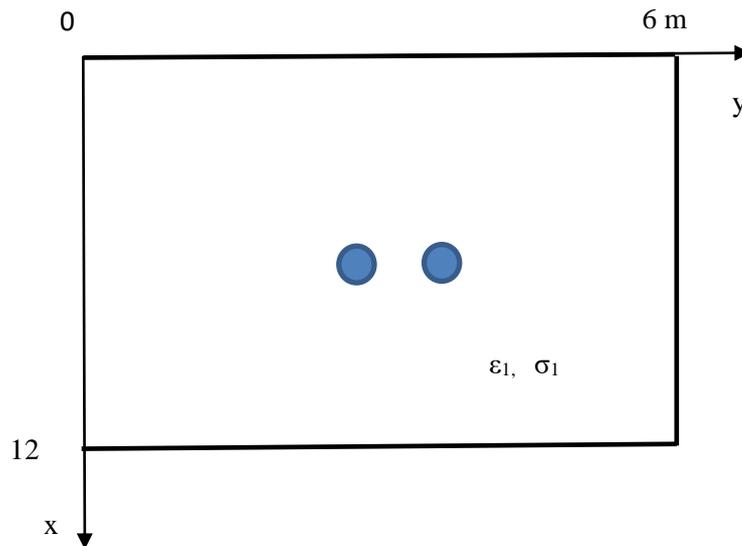


Figure 15 - Schematic of an experiment with a georadar with two inhomogeneities

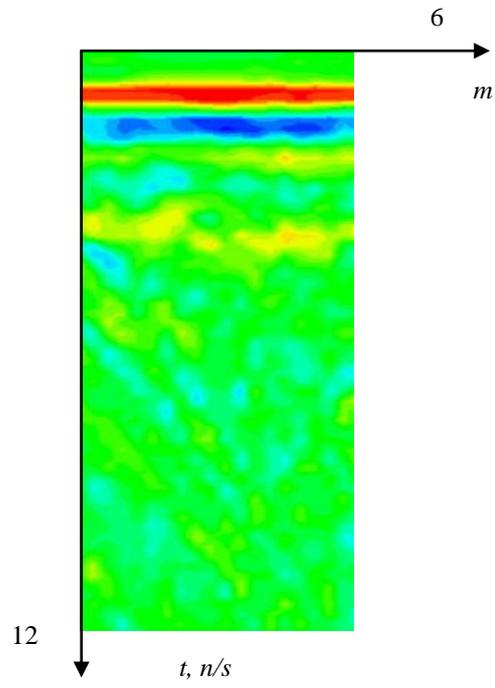


Figure 16 - Summarized data from a georadar with two inhomogeneities

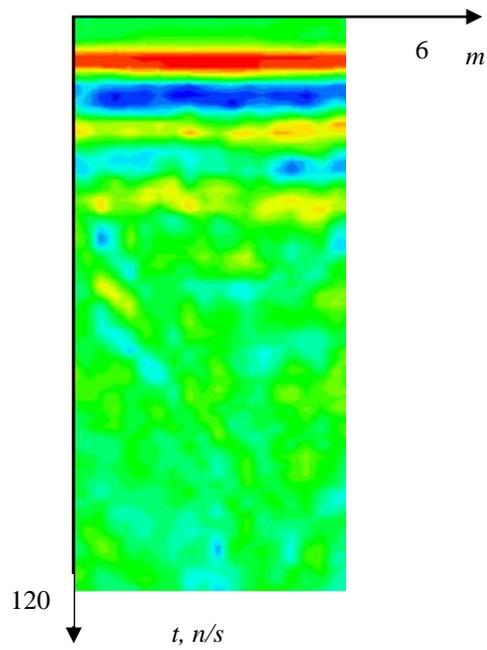


Figure 17 - Summarized georadar data without heterogeneity

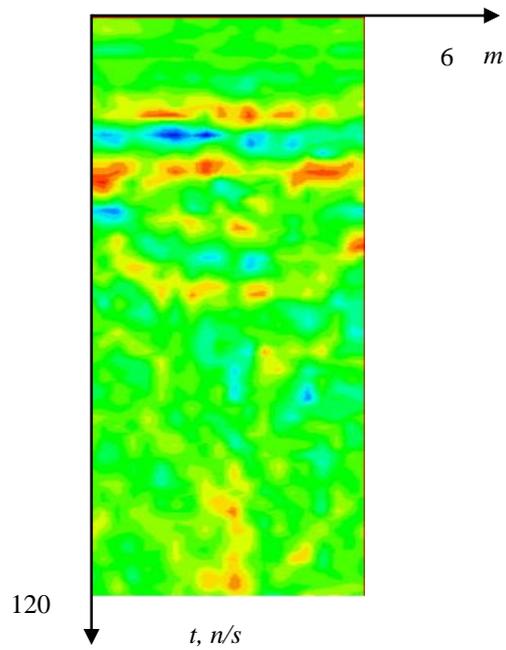


Figure 18 - Calculated difference of combined radar data without inclusion and with two inhomogeneity

The obtained data are similar to the numerically anomalous field obtained at the boundary $x = 0$ m. The results of processing the radargrams improve the detection of localized objects. But it is clear that the inhomogeneity are very fuzzy and weak. This proves that the continuation of the field in the direction of inhomogeneity would help to reveal the number of inhomogeneity more accurately.

CONCLUSION

The research work is devoted to theoretical, numerical and applied analysis of the solution of direct and inverse problems for the equation of electrodynamics (near-surface radar), as well as to the theoretical investigation of the developed algorithms, approbation of methods for interpreting the data of the Loza-V georadar.

In the research work the following results were obtained:

- The algorithm for numerical solution of the inverse problem for the geo-electric equation in a Cartesian coordinate system is modified.
- The numerical solution of the geo-electric equation in the Cartesian coordinate system is analyzed, and the results qualitatively coincide with the georadar data. Criteria for the sensitivity of the electromagnetic probe in the well with respect to changes in the parameters of the host medium and the penetration zone have been obtained.
- Theoretical and numerical studies were carried out to solve the problem of continuation of electromagnetic fields.
- The continuation problem is reduced to the inverse of some direct problem.
- An analysis of the numerical solution of the inverse problem in the presence of several localized objects is carried out. The results of comparison of numerical calculations and georadar data show that the solution of the problem of continuation of the electromagnetic field to a depth h in the direction of inhomogeneity can distinguish heterogeneities and clarify their number.

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