ТЕПЛОВОЙ ПОГРАНИЧНЫЙ СЛОЙ

Толщина динамического пограничного слоя δ_d $\quad \dfrac{\delta_{\ddot{a}}}{L} \sim \dfrac{1}{\sqrt{\mathrm{Re}}}$

Найдем выражение для толщины теплового пограничного слоя $\boldsymbol{\delta_T}$

Из уравнения энергии

теплопроводность $\dfrac{\partial}{\partial y}\lambda\dfrac{\partial T}{\partial y}$

конвекция $\rho C_p u \dfrac{\partial T}{\partial x}$

В температурном пограничном слое

↓

процессы теплопроводности и конвекции соизмеримы

$$\frac{\partial}{\partial y}\lambda\frac{\partial T}{\partial y} \approx \rho C_p u \frac{\partial T}{\partial x}$$

Запишем через характерные размеры

$$\Rightarrow \frac{\lambda T_\infty}{\delta_T^2} \approx \rho C_p \frac{U_\infty}{L} T_\infty \cdot \frac{\nu}{\nu} \cdot \frac{L}{L}$$

$$\frac{1}{\delta_T^2} \approx \frac{\rho C_p \nu}{\lambda} \cdot \frac{U_\infty L}{\nu} \cdot \frac{1}{L^2} \approx \boldsymbol{Pr\ Re}\,\frac{1}{L^2} \Rightarrow \frac{\delta_T^2}{L^2} \approx \frac{1}{\mathrm{Pr}}\,\frac{1}{\mathrm{Re}} \Rightarrow \bar{\delta}_T \approx \frac{1}{\sqrt{Pr}}\,\frac{1}{\sqrt{Re}}$$

$$\bar{\delta}_{\ddot{a}} \sim \frac{1}{\sqrt{\mathrm{Re}}} \Rightarrow \frac{\bar{\delta}_T}{\bar{\delta}_{\ddot{a}}} \sim \frac{1}{\sqrt{Pr}} \Rightarrow \frac{\delta_T}{\delta_{\ddot{a}}} \sim \frac{1}{\sqrt{\mathrm{Pr}}}$$

Для газов Pr~1 $\Rightarrow \delta_T \approx \delta_{\ddot{a}}$

<u>При T = 300 K^0</u>

для гелия Pr=0.68,

для аргона число Pr=0.77,

для водорода Pr=0.69.

$$\mathbf{Pr}\uparrow \Rightarrow \delta_{\text{Т}} < \delta_{\partial}$$

В жидкостях $\delta_{\text{Т}} < \delta_{\partial}$, так как число Прандтля для жидкостей Pr>1

<u>При T = 23^{0}C</u>

для воды H_2O Pr=7

для смазочного масла Pr=677

ПОЛУЧЕНИЕ УРАВНЕНИЯ ДЛЯ ТЕМПЕРАТУРНОГО ПОГРАНИЧНОГО СЛОЯ

Запишем уравнение энергии в дивергентной форме:

$$\frac{\partial}{\partial x_k}\left\{\rho\upsilon_k\left(i+\frac{\upsilon^2}{2}\right)-\mu\frac{\partial}{\partial x_i}\left(\frac{i}{Pr}+\upsilon^2\right)+\mu[\vec{\upsilon}\times rot\,\vec{\upsilon}]\right\}=0$$

$$\upsilon^2=u^2+\upsilon^2+w^2$$

$$\frac{\partial}{\partial x}\left\{\rho u\left(i+\frac{u^2+\upsilon^2}{2}\right)-\mu\frac{\partial}{\partial x}\left(\frac{i}{Pr}+u^2+\upsilon^2\right)+\mu[\vec{\upsilon}\times rot\,\vec{\upsilon}]_x\right\}+$$

$$+\frac{\partial}{\partial y}\left\{\rho\upsilon\left(i+\frac{u^2+\upsilon^2}{2}\right)-\mu\frac{\partial}{\partial y}\left(\frac{i}{Pr}+u^2+\upsilon^2\right)+\mu[\vec{\upsilon}\times rot\,\vec{\upsilon}]_y\right\}=0 \quad (1)$$

$$[\vec{\upsilon}\times rot\vec{\upsilon}]_x=\begin{vmatrix} i & j & k\\ u & \upsilon & w\\ rot_x & rot_y & rot_z\end{vmatrix}=\upsilon rot_z\vec{\upsilon}-w rot_y\vec{\upsilon}=\upsilon rot_z\vec{\upsilon}$$

(зачёркнуто: $w\,rot_y\vec{\upsilon}$ → **0**)

$$rot_z\vec{\upsilon} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ u & \upsilon & w \end{vmatrix} = \left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right)$$

δ

$$\left[\vec{\upsilon}\times rot\vec{\upsilon}\right]_x = -\upsilon\frac{\partial u}{\partial y}$$

δ

$$\left[\vec{\upsilon}\times rot\vec{\upsilon}\right]_y = w\,rot_x\vec{\upsilon} - u\,rot_z\vec{\upsilon} = -u\,rot_z\vec{\upsilon} = -u\left(\frac{\partial \upsilon}{\partial x} - \frac{\partial u}{\partial y}\right) = u\frac{\partial u}{\partial y}$$

$1 \quad 1 \quad 1 \quad 1 \quad \delta^2 \quad \delta^2 \quad 1 \quad 1 \quad \delta^2 \quad \delta^2 \quad \delta \quad 1/\delta$

(1) $\Rightarrow$

$$\frac{\partial}{\partial x}\left\{\rho u\left(i + \frac{u^2+\upsilon^2}{2}\right) - \mu\frac{\partial}{\partial x}\left(\frac{i}{Pr} + u^2 + \upsilon^2\right) - \mu\upsilon\frac{\partial u}{\partial y}\right\} +$$

$$+\frac{\partial}{\partial y}\left\{\rho\upsilon\left(i + \frac{u^2+\upsilon^2}{2}\right) - \mu\frac{\partial}{\partial y}\left(\frac{i}{Pr} + u^2 + \upsilon^2\right) + \mu u\frac{\partial u}{\partial y}\right\} = 0$$

$1/\delta \quad 1 \quad \delta \quad 1 \quad 1 \quad \delta^2 \quad \delta^2 \quad 1/\delta \quad 1 \quad 1 \quad \delta^2 \quad \delta^2 \quad 1 \quad 1/\delta$

$$\frac{\partial}{\partial x}\left(\rho u\left(i+\frac{u^2}{2}\right)\right)+\frac{\partial}{\partial y}\left\{\rho \upsilon\left(i+\frac{u^2}{2}\right)-\mu\frac{\partial}{\partial y}\left(\frac{i}{Pr}+u^2\right)+\mu u\frac{\partial u}{\partial y}\right\}=0$$

$$i+\frac{u^2}{2}=i_0$$

$$\frac{\partial}{\partial x}\left(\rho u i_0\right)+\frac{\partial}{\partial y}\left(\rho \upsilon i_0\right)-\frac{\partial}{\partial y}\mu\frac{\partial}{\partial y}\left(\frac{i}{\mathrm{Pr}}+u^2\right)+\frac{\partial}{\partial y}\mu u\frac{\partial u}{\partial y}=0$$

$$\frac{i}{\mathrm{Pr}}+u^2=i+\frac{u^2}{2}-i+\frac{u^2}{2}+\frac{i}{\mathrm{Pr}}=i_0+i\left(\frac{1}{\mathrm{Pr}}-1\right)+\frac{u^2}{2}$$

$$-\frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial y}\left(\frac{i}{\mathrm{Pr}}+u^2\right)\right)=-\frac{\partial}{\partial y}\left(\mu\frac{\partial}{\partial y}i_0+\mu\frac{\partial}{\partial y}i\left(\frac{1}{\mathrm{Pr}}-1\right)+\mu\frac{\partial}{\partial y}\left(\frac{u^2}{2}\right)\right)$$

$$\frac{\partial}{\partial x}\left(\rho u i_0\right)+\frac{\partial}{\partial y}\left(\rho \upsilon i_0\right)-\frac{\partial}{\partial y}\mu\frac{\partial i_0}{\partial y}-\frac{\partial}{\partial y}\mu\left(\frac{1}{Pr}-1\right)\frac{\partial i}{\partial y}-\frac{\partial}{\partial y}\mu u\frac{\partial u}{\partial y}+\frac{\partial}{\partial y}\mu u\frac{\partial u}{\partial y}=0$$

$$\frac{\partial}{\partial x}\left(\rho u i_0\right)+\frac{\partial}{\partial y}\left(\rho \upsilon i_0\right)-\frac{\partial}{\partial y}\mu\frac{\partial i_0}{\partial y}-\frac{\partial}{\partial y}\mu\left(\frac{1}{Pr}-1\right)\frac{\partial i}{\partial y}=0$$

$$\frac{\partial}{\partial x}\left(\rho u i_0\right)+\frac{\partial}{\partial y}\left(\rho \upsilon i_0\right)=\frac{\partial}{\partial y}\left\{\mu\frac{\partial i_0}{\partial y}+\mu\left(\frac{1}{Pr}-1\right)\frac{\partial i}{\partial y}\right\}$$

$$\frac{\partial}{\partial x}\left(\rho u i_0\right)=i_0\frac{\partial}{\partial x}\left(\rho u\right)+\rho u\frac{\partial i_0}{\partial x}$$

+

$$\frac{\partial}{\partial y}\left(\rho \upsilon i_0\right)=i_0\frac{\partial}{\partial y}\left(\rho \upsilon\right)+\rho \upsilon\frac{\partial i_0}{\partial y}$$

$$i_0\frac{\partial}{\partial x}\left(\rho u\right)+\rho u\frac{\partial i_0}{\partial x}+i_0\frac{\partial}{\partial y}\left(\rho \upsilon\right)+\rho \upsilon\frac{\partial i_0}{\partial y}=i_0\left(\frac{\partial}{\partial x}\left(\rho u\right)+\frac{\partial}{\partial y}\left(\rho \upsilon\right)\right)+\rho u\frac{\partial i_0}{\partial x}+\rho \upsilon\frac{\partial i_0}{\partial y}$$

→ 0

$$\rho u\frac{\partial i_0}{\partial x}+\rho \upsilon\frac{\partial i_0}{\partial y}=\frac{\partial}{\partial y}\left\{\mu\frac{\partial i_0}{\partial y}+\mu\left(\frac{1}{Pr}-1\right)\frac{\partial i}{\partial y}\right\}$$

→ **уравнение энергии для полной энтальпии для теплового погранслоя**

$$i_0 = i + \frac{u^2}{2}$$

$$\rho u \frac{\partial}{\partial x}\left(i + \frac{u^2}{2}\right) + \rho \upsilon \frac{\partial}{\partial y}\left(i + \frac{u^2}{2}\right) = \frac{\partial}{\partial y}\left\{\mu \frac{\partial}{\partial y}\left(i + \frac{u^2}{2}\right) + \mu\left(\frac{1}{Pr} - 1\right)\frac{\partial i}{\partial y}\right\}$$

$$\rho u \frac{\partial}{\partial x}\left(i + \frac{u^2}{2}\right) + \rho \upsilon \frac{\partial}{\partial y}\left(i + \frac{u^2}{2}\right) = \frac{\partial}{\partial y}\left\{\mu \frac{\partial i}{\partial y} + \mu u \frac{\partial u}{\partial y} + \mu \frac{1}{Pr}\frac{\partial i}{\partial y} - \mu \frac{\partial i}{\partial y}\right\}$$

$$\rho u \frac{\partial i}{\partial x} + \rho u u \frac{\partial u}{\partial x} + \rho \upsilon \frac{\partial i}{\partial y} + \rho \upsilon u \frac{\partial u}{\partial y} = \frac{1}{Pr}\frac{\partial}{\partial y}\left(\mu \frac{\partial i}{\partial y}\right) + \frac{\partial}{\partial y}\mu u \frac{\partial u}{\partial y} \qquad (2)$$

Уравнение движения: $$\rho u \frac{\partial u}{\partial x} + \rho \upsilon \frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right)$$

умножим на *u*

$$\rho u u \frac{\partial u}{\partial x} + \rho \upsilon u \frac{\partial u}{\partial y} = -u \frac{\partial p}{\partial x} + u \frac{\partial}{\partial y}\mu \frac{\partial u}{\partial y}$$

(2) ⇒ $$\rho u \frac{\partial i}{\partial x} + \rho \upsilon \frac{\partial i}{\partial x} - u \frac{\partial p}{\partial x} + u \frac{\partial}{\partial y}\mu \frac{\partial u}{\partial y} = \frac{1}{Pr}\frac{\partial}{\partial y}\left(\mu \frac{\partial i}{\partial y}\right) + \frac{\partial}{\partial y}\mu u \frac{\partial u}{\partial y} \qquad (3)$$

$$\frac{\partial}{\partial y}\mu u\frac{\partial u}{\partial y} = u\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

Подставим в (3)

$$\rho u\frac{\partial i}{\partial x} + \rho\upsilon\frac{\partial i}{\partial x} - u\frac{\partial p}{\partial x} + u\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y} = \frac{1}{Pr}\frac{\partial}{\partial y}\left(\mu\frac{\partial i}{\partial y}\right) + u\frac{\partial}{\partial y}\mu\frac{\partial u}{\partial y} + \mu\left(\frac{\partial u}{\partial y}\right)^2$$

$$\rho u\frac{\partial i}{\partial x} + \rho\upsilon\frac{\partial i}{\partial y} - u\frac{\partial p}{\partial x} = \mu\left(\frac{\partial u}{\partial y}\right)^2 + \frac{1}{Pr}\frac{\partial}{\partial y}\left(\mu\frac{\partial i}{\partial y}\right)$$

- уравнение энергии для теплового пограничного слоя

Конвективный перенос энергии

Работа сил сжатия

Работа сил трения

Теплопроводность

Мы должны задать граничные условия

Температура стенки: $i_0 = i_w(x), \quad T_0 = T_w(x)$

Или задать поток тепла: $q = q_w(x) = -\lambda \frac{\partial T}{\partial y}\Big|_w$

На верхней границе пограничного слоя можно задать: $i(\infty) = i_\infty, \quad T(\infty) = T_\infty$

Для несжимаемой жидкости (*ρ=const*)
с постоянными свойствами (*μ = const*) ⇒ **можно пренебречь теплотой трения** ⇒

$$u\frac{\partial u}{\partial x} + \upsilon\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial \upsilon}{\partial y} = 0$$

$$u\frac{\partial T}{\partial x} + \upsilon\frac{\partial T}{\partial y} = a\frac{\partial^2 T}{\partial y^2},$$

где $a = \frac{\lambda}{\rho C_p}$

коэффициент теплоотдачи

Система уравнений пограничного слоя