Lecture no.9

Transport Properties of Nonideal Plasmas (continuation)

Introduction

The equation derived in the previous lecture describes the electron mobility on the basis of the electron-atom collision frequency and is not valid for the case when the electron–atom collision frequency ν becomes comparable with the electron–ion collision frequency ν_i .

Electrical conductivity of strongly ionized plasma

The above mentioned situation was considered by Spitzer (1962). It is known that due to the long–range character of the Coulomb interaction a small-angle scattering is important in strongly ionized plasma. The small-angle scattering can be taken into account by so-called "Coulomb logarithm" $\ln \Lambda$. This quantity in a classical plasma is defined as the ratio between the Debye radius r_D and the Landau length $Ze^2 / k_B T$. More information can be found in the Table 1 (see, below). According to Spitzer we can write the expression for electron-ion collision frequency in the following form:

$$\nu_i(E) = 2\pi e^4 Z^2 E^{-3/2} (2m)^{-1/2} n_i \ln \Lambda \,. \quad (1)$$

If we assume that $\ln \Lambda$ does not depend on energy then averaging over energies gives the following formula for electron-ion collision frequency:

$$\nu_i = \frac{\pi^{3/2}}{4\sqrt{2}} n_i \sqrt{\frac{k_B T}{m}} \left(\frac{Ze^2}{k_B T} \right)^2 \ln \Lambda \,. \quad (2)$$

Using the electro-neutrality condition $n_e = Zn_i$ we get the expression for electrical conductivity:

$$\sigma = \frac{2(2k_B T)^{3/2}}{\pi^{3/2} Z e^2 m^{1/2}} \frac{1}{\ln\Lambda} \,, \tag{3}$$

where $\ln\Lambda = \ln\frac{3}{\gamma}$; $\gamma = Ze^2/(r_D k_B T)$ is the nonideality parameter; $r_D = (4\pi n_e e^2/k_B T)^{-1/2}$ is the Debye radius of electrons. In plasma due to the Coulomb long–range interaction the electron–electron correlations influence on the electrical conductivity even at small values of nonideality parameter. In order to take into account these correlations, the term which is responsible for electron–electron collisions must be added to the right–hand side of the kinetic equation.

It should be noted that the electron–electron interactions cause a decreasing of the electrical conductivity. Initially the velocity distribution function of electrons is spherically symmetric. The applied field disturbs this symmetrical distribution and the electron–electron interactions oppose to this disturbance, consequently, the mobility of electrons decreases. Thus the resulting expression for the electrical conductivity of fully ionized plasma (Spitzer's formula) is

$$\sigma_{Sp} = C(Z)\cdot\frac{2(2k_B T)^{3/2}}{\pi^{3/2} Z e^2 m^{1/2}} \frac{1}{\ln\Lambda} \,, \tag{4}$$

where $C(Z) = 0{,}582 \div 1{,}0$ at different values of charge number Z, i.e. C(1)=0,582 and $C(\infty) = 1{,}0$. For singly charged ions ($Z = 1$) we have

$$\sigma_{Sp} = 1{,}53\cdot 10^{-4}\cdot\frac{T^{3/2}}{\ln\Lambda}\ Ohm^{-1}\cdot cm^{-1} \,, \tag{5}$$

where T is the temperature in K. The Spitzer theory is valid for classical weakly nonideal plasma.

The Coulomb logarithm for a plasma

As it was above mentioned, the Coulomb logarithm is an important factor in the kinetic theory of plasma. In the general case it can be defined as a ratio between maximal and minimal values of impact parameter, i.e. $\Lambda = b_{\max}/b_{\min}$. It should be noted that the Coulomb logarithm has different expressions in the cases of classical and quantum plasmas (see, Table 1).

Table 1. Classical and quantum expressions for the Coulomb logarithm.

Typical lengths of Coulomb interactions	Classical plasma		Quantum plasma	
	$r_D > n_e^{-1/3}$	$r_D < n_e^{-1/3}$	$r_D > n_e^{-1/3}$	$r_D < n_e^{-1/3}$
$b_{\max}$	$\sim r_D$	$\sim a \approx n_e^{-1/3}$	$\sim r_D$	$\sim a \approx n_e^{-1/3}$
$b_{\min}$	$\sim e^2/k_B T$	$\sim e^2/k_B T$	$\sim \lambdabar_e$	$\sim \lambdabar_e$
Λ	$\sim \Gamma^{-3/2}$	$\sim \Gamma^{-1}$	$\sim \Gamma^{-3/2} \cdot T^{-1/2}$	$\sim \Gamma^{-1} \cdot T^{-1/2}$

According to the Table 1 we can do the following conclusions:

- For the classical plasma, when $e^2/k_B T >> \lambdabar_e$, then $\Lambda = f(\Gamma)$ and we have $\sigma^* = f(\Gamma)$, here $\sigma^* = e^2 m^{1/2} (k_B T)^{-3/2} \cdot \sigma$. Consequently, in the classical regime the isobars and isotherms for different matters are described by the universal curve $\sigma^*(\Gamma)$ on the coordinate plane $\sigma^* - \Gamma$. Therefore, we can conclude that the Coulomb properties of plasma have a similarity in this case.
- The similarity is not realized when the quantum effects become essential at $b_{\min} \sim \lambdabar_e$ and in this case we have the additional dependence of the Coulomb logarithm on a temperature. This fact can explain the layering (stratification) of isotherms on $\sigma^* - \Gamma$ diagram observed experimentally.

The electrical conductivity of nonideal plasma. The Chapman-Enskog method

The transport properties of plasma can be calculated by kinetic equation method. In this case it is necessary to know the data of scattering processes of particles or the interaction potential between particles. For instance, the electrical conductivity of plasma is defined as follows:

$$\sigma = \frac{3e^2}{8m\Omega^{(1)}(1)} \quad , \tag{6}$$

where

$$\Omega^{(l)}(r) = \sqrt{\pi}\int_0^\infty \exp(-\vec{g}^2)\vec{g}^{(2r+2)}\Phi^{(l)}(\vec{g})d\vec{g}\,;$$
$$\Phi^{(l)}(\vec{g}) = 2(k_B T/m)^{1/2}\vec{g}\int_0^\infty b\left[1-\cos^l \vartheta(b,\vec{g})\right]db\,; \tag{7}$$

b is the impact parameter; $\vec{g} = \vec{u}/2(k_B T/m)^{-1/2}$ is the dimensionless relative velocity.

The resulting isotherms of the electrical conductivity are shown in Fig. 1 as function of the total electron density. Comparison with the fully ionized case shows, that partial ionization leads to a lowering of the electrical conductivity, especially for low temperatures. This may lead to a pronounced minimum in the conductivity isotherms.

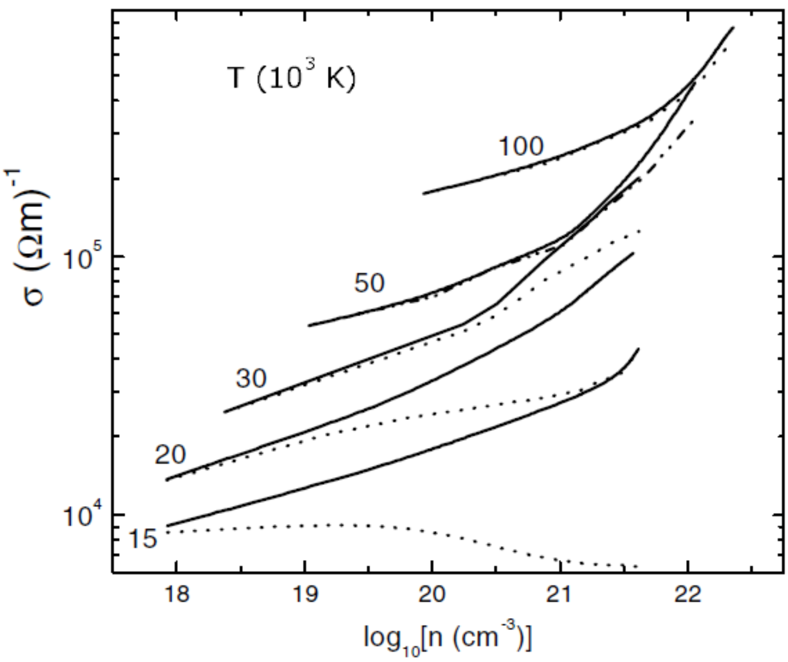

Figure 1. Electrical conductivity of partially ionized hydrogen plasma (broken lines) in comparison with the results for the fully ionized case (solid lines) for various temperatures as function of the total electron density.

The dimensionless electrical conductivity σ^* is shown in Fig. 2 as a function of the nonideality parameter γ. For ideal plasmas $\gamma << 1$, we have good agreement with the Spitzer theory. The results for the fully ionized case agree well with the data for the partially ionized plasma for $\gamma \leq 0{,}1$. For higher nonideality parameters, the electrical conductivity of the partially ionized plasma is substantially lower. In the strong coupling regime $\gamma \sim 1$ we have good agreement with the experimental data.

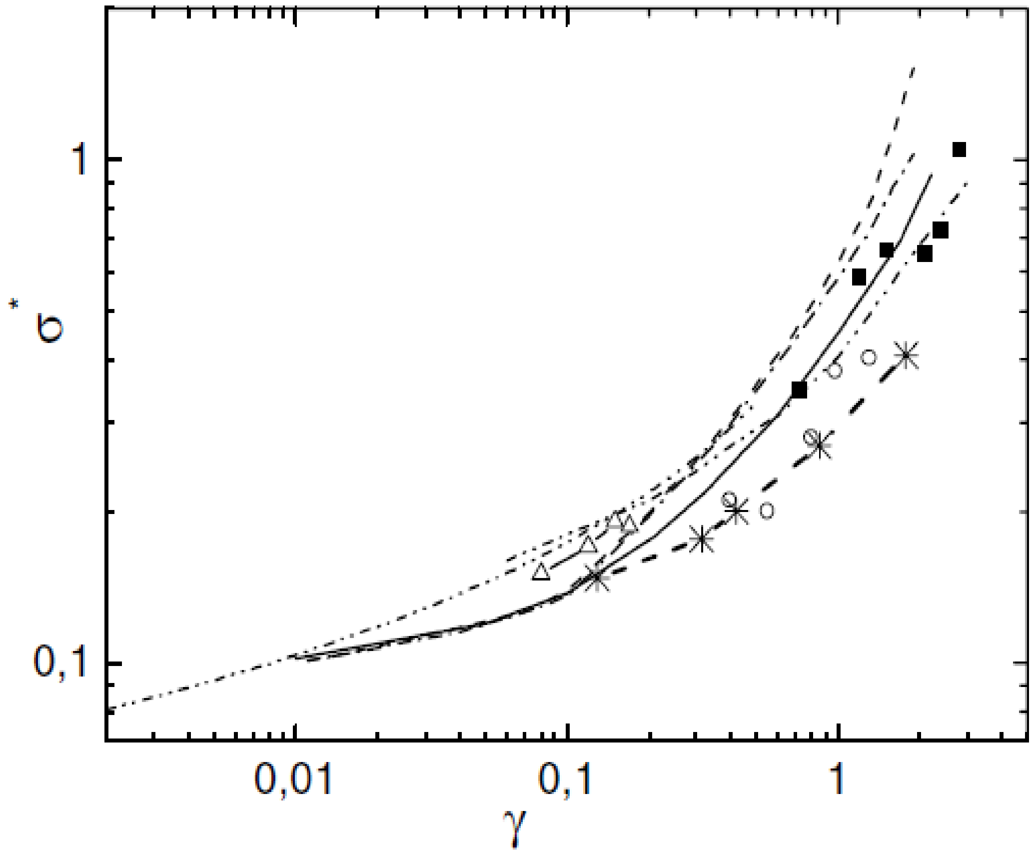

Figure 2. Reduced electrical conductivity as a function of the nonideality parameter. Full line represents the results for partially ionized plasma (T.Ramazanov e.a.); dashed and dash-dotted lines denote the data for fully ionized plasma (Kh.Nurekenov & S.Kodanova); dotted line is the Spitzer theory; asterisks are results of Ichimaru; triangles, boxes and circles represent the experimental data (Radtke, Ivanov e.a.).