

REPORT

supervisor on the PhD dissertation
al-Farabi Kazakh National University and
Institute of Mathematics and Mathematical Modeling
Sabitbek Bolys Mazhituly

On theme: «**Hardy-Sobolev type inequalities on homogeneous groups and applications**», submitted for the degree of Doctor of Philosophy (PhD) in the speciality "6D060100 - Mathematics"

The thesis of Sabitbek Bolys "Hardy-Sobolev type inequalities on homogeneous groups and applications" is devoted to the study of subelliptic functional inequalities on homogeneous groups.

The PhD dissertation has four directions for investigating subelliptic functional inequalities on homogeneous groups, such as geometric functional inequalities, horizontal subelliptic functional inequalities, subelliptic Hardy and Rellich inequalities with the sub-Laplacian fundamental solutions and weighted Hardy and Sobolev inequalities for general vector fields without group structure.

In the first direction, which is the geometric functional inequality, the hypothesis about the natural weight for the geometric Hardy inequality by Larson was solved. Also, the geometrical Hardy-Sobolev and the Hardy inequalities on starshaped sets are obtained on the Heisenberg group and stratified groups, respectively.

In the second direction, interesting results are obtained as analogues of inequalities of Hardy type with multiple singularities and many-particle Hardy inequality.

In the third direction, he generalized the weight inequalities of the Hardy, Rellich, and Caffarelli-Con-Nirenberg type with boundary terms obtained on stratified Lie groups. As a result, most Hardy-type inequalities and uncertainty principles on the stratified groups are recovered.

In the last direction, the weighted Hardy and Rellich inequalities are established for general vector fields without group structure.

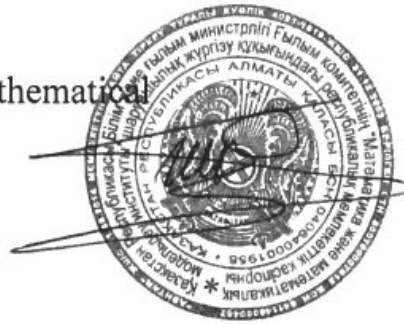
Turning to the assessment of the PhD thesis as a whole, it should be noted that the author of the dissertation was able to establish a number of new, interesting results on homogeneous Lie groups. The main results of the thesis are formulated as rigorously proved theorems. They are reliable and reasonable. Auxiliary statements of the problematic issues of each section are formulated in the form of lemmas and statements, and they are also rigorously proved.

Note that the doctoral candidate has a sufficiently general scientific methodology, logic and technology for conducting research.

The results of the thesis were published in 15 papers, including 4 articles in foreign journals that have a non-zero impact factor according to the Tomson Reuters scientific journals database, 3 articles from the list recommended by the Committee on the Control of Education and Science of the MES RK, 8 works in materials international conferences.

I consider that in its scientific significance and novelty of the obtained results, the dissertation work of Sabitbek Bolys on the topic "Hardy-Sobolev type inequalities on homogeneous groups and applications" satisfies all the requirements of PhD dissertations presented to the doctor's degree and its author deserves the degree of Doctor of Philosophy (PhD) in the speciality "6D060100 - Mathematics".

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T.Sh. Kalmenov