

## REVIEW

**on the thesis of Tatyana Zambarnaya  
“Countable models of small dependent theories”  
submitted for the degree of Doctor of Philosophy  
(PhD) in the specialty “6D060100 – Mathematics”**

### **1. The relevance of the research topic and its relationship with general scientific and national programs.**

One of the main open problems in model theory is the Vaught's conjecture. It was proposed by R. Vaught in 1961, and states that every complete countable first-order theory has either finite number, or  $\aleph_0$ , or  $2^{\aleph_0}$  countable models up to an isomorphism. Taking into account Morley's result, the conjecture can be reformulated as whether such a theory can have  $\aleph_1$  countable nonisomorphic models ( $\aleph_0 < \aleph_1 < 2^{\aleph_0}$ ). The Vaught hypothesis has been proved only for particular classes of theories, among them uncountably categorical,  $\omega$ -stable, o-minimal and quite o-minimal theories.

T. Zambarnaya studies the number of countable models for small dependent (NIP) theories. Note that non-small theories have  $2^{\aleph_0}$  countable models and therefore are not of an interest from the point of view of the Vaught's conjecture. She found properties with which a theory has maximal number of countable nonisomorphic models, which reduces the class of theories which should be investigated to those without the obtained properties; as well as proved the conjecture for a particular subclass of dependent theories.

All of the above indicates the relevance of the research topic of the dissertation.

The dissertation was prepared within the scientific projects of the program of grant financing of fundamental researches in the area of natural sciences of the Ministry of education and science of the Republic of Kazakhstan “Properties of types in dependent theories” (2015-2017 years, 5125/GF4) and “Conservative extensions, countable ordered models and closure operators” (2018-2020 years, AP05134992).

### **2. Scientific results and their validity.**

In the dissertation, the following main results were obtained:

- it has been proved that a countable linearly ordered theory which has a non-principal extremely trivial 1-type over a finite subset, then the theory has  $2^{\aleph_0}$  countable non-isomorphic models;
- it has been proved that if there exists a formula determining a partial order on tuples such that there exists a finite discrete chain of length greater or equal to any given natural number, then a countable theory has  $2^{\aleph_0}$  countable non-isomorphic models;
- it has been proved that countable linearly ordered theory which has a quasi-successor formula on a non-principal 1-type, then this theory has  $2^{\aleph_0}$  countable non-isomorphic models;
- it has been proved that the class of weakly o-minimal theories of convexity rank 1 satisfies the Vaught's conjecture.

All of these results are theoretically justified and supplied with rigorous mathematical proofs.

### **3. The degree of validity and reliability of each scientific result (scientific position), proofs and conclusion of the applicant formulated in the thesis.**

All the results, proofs and conclusion are scientifically justified. All presented proofs are complete and valid. The obtained results were approbated on mathematical

conferences and seminars, and were published in scientific journals including Scopus and Web of Science indexed journals, such as Siberian Electronic Mathematical Reports, and Annals of Pure and Applied Logic. This guarantees validity of the results presented in the dissertation.

**4. The degree of novelty of each scientific result (position), the conclusion of the applicant formulated in the thesis.**

All results obtained by the applicant are new, and have an undoubted scientific interest. The results extend known results of model theory specialists.

**5. Practical and theoretical significance of scientific results.**

All the results are of a theoretical nature and can be applied in further study of countable spectrum of small theories. The results on maximality of the number of countable models constitute properties, which a counterexample to the Vaught conjecture does not have, and therefore are of an importance to the study of existence of such a counterexample.

**6. Comments, suggestions for the thesis.**

1. On 1<sup>st</sup> page instead “For” should be “for”.
2. Need to choose one style with capital letters, since on second page sometimes “...Omitting types”, but sometimes “Main Kinds...”.
3. Misusing of articles.
4. On 8<sup>th</sup> page, 1<sup>st</sup> paragraph, 5<sup>th</sup> line instead “problems is ...” should be “problem is ...”; 2<sup>nd</sup> paragraph, 4<sup>th</sup> line instead “there exist ...” should be “there exists ...”; 4<sup>th</sup> paragraph 3<sup>rd</sup> line instead “that is equal ...” should be either “that is equal to...” or “that equals...”.
5. On 9<sup>th</sup> page, 13<sup>th</sup> line instead “is greater or ...” should be “is greater than or ...”.
6. It is necessary to correct a large number of such types of misprints that occur throughout the thesis.
7. It is necessary to reformulate some sentences in English.

These comments are not related to the content of the thesis and do not reduce the quality of the thesis as a whole.

**7. Compliance with the content of the thesis in the framework of the requirements of the Rules for the award of scientific degrees.**

The thesis was performed at a high scientific level; it is a complete scientific work. The content of the thesis “Countable models of small dependent theories” meets all the requirements for the degree of Doctor of Philosophy (PhD), and its author, Zambarnaya Tatyana Sergeevna, deserves the award of the degree of Doctor of Philosophy (PhD) on specialty 6D060100 – “Mathematics”.

**Official reviewer,  
PhD, Professor of Kazakh-British  
Technical University**



**A.A. Issakhov**