

REVIEW

on the PhD thesis of Sabitbek Bolys Mazhituly «*Hardy-Sobolev type inequalities on homogeneous groups and applications*», submitted for the degree of Doctor of Philosophy (PhD) in the speciality "6D060100 - Mathematics"

1. The relevance of the research topic and its relationship with general scientific and national programs.

The main goal of this PhD thesis is to construct theory for subelliptic functional inequalities and qualitative research of their applications. To achieve this aim, B.M. Sabitbek developed existent and created new analytic methods of analysis on nilpotent Lie groups, then to study general subelliptic operators. This study led to a deeper understanding of the basic Lie group structure of functional inequalities and functional spaces for subelliptic operators and serve as the foundation of a new mathematical theory with applications in various areas of modern mathematics and theoretical physics.

The relevance of PhD thesis in this field of knowledge is confirmed with the publications in the high-ranking scientific journals.

2. Scientific results and their validity.

The PhD thesis consists of Introduction, four chapters, Conclusion and References.

As the main result of the first chapter can be attributed as follows

- Obtaining the geometric Hardy-Sobolev inequality on the half-space of the Heisenberg, which is new inequality in the subelliptic setting;
- Proving the conjecture about the weight function for the geometric Hardy inequality on the half-space of the Heisenberg group;
- Establishing the Hardy inequality on starshaped sets of the stratified groups.

As the main result of the second chapter can be attributed as follows

- Obtaining the embedding theorem on the stratified groups by using the horizontal Hardy-Rellich type inequalities;
- Establishing the various type of uncertainty principles as the consequence of the horizontal Hardy-Rellich type inequalities;
- Proving the anisotropic Hardy and Rellich inequalities on the stratified groups.

As the main result of the third chapter can be attributed as follows:

- Obtaining L_p -Hardy type inequalities with weight functions and boundary terms, as the consequence Badiale-Tarantello conjecture is proved.
- Establishing L_p -Rellich type inequalities with weight functions.
- Proving Hardy and Rellich type inequalities on the quaternion Heisenberg group.

As the main result of the fourth chapter can be attributed as follows:

- Obtaining weighted anisotropic Hardy type inequality for general vector fields;

- Establishing weighted anisotropic Rellich type inequality for general vector fields.

3. The degree of validity and reliability of each scientific result, proofs and conclusions of the applicant formulated in the thesis.

All obtained results, proofs and conclusion of the applicant are reliable and provided with detail evidence. All statements showed in the PhD thesis have a correct theoretical substantiation. The results obtained by the applicant have been presented in the international conferences and scientific seminars.

4. The degree of novelty of each scientific result, proof of the applicant formulated in the PhD thesis.

In the PhD dissertation, all established results are new and interesting.

Most of the results have been published in a peer-reviewed scientific journals by the Web of Science and Scopus with a non-zero impact factor: Complex Analysis and Operator Theory, Mathematical Modeling and Natural Phenomen, Revista Matematica Complutense, Nonlinear Differential Equations and Applications.

5. Practical and theoretical significance of scientific results.

The results obtained in the PhD thesis is the fundamental research and can be applied to the subelliptic PDE, subelliptic spectral theory and etc. The results of this PhD thesis can also be used in the preparation and reading of special courses for undergraduates and graduate students in the mathematics.

6. Comments, suggestions for the PhD thesis.

In the text of the PhD thesis, there are some typos and stylistic inaccuracies that need to be corrected. These comments do not affect the positive assessment of the work as a whole.

7. Compliance with the content of the PhD thesis in the framework of the requirements of the Rules for the award of scientific degrees.

The PhD thesis was performed at a high scientific level; it is a complete scientific work. The content of the PhD thesis on "Hardy-Sobolev type inequalities on homogeneous groups and applications" meets all the requirements for the thesis for the degree of Doctor of Philosophy (PhD), and its author, B.M. Sabitbek, deserves the award of the degree of Doctor of Philosophy (PhD) on specialty «6D060100-Mathematics».

Official Reviewr

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РАСТАЙМЫН

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ЗАВЕРЯЮ

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