

## REVIEW

of scientific supervisor, doctor of physical and mathematical sciences, corresponding member of the National Academy of Sciences of Kazakhstan Baizhanov B.S. on dissertation of Tatyana Sergeevna Zambarnaya on topic "Countable Models of Small Dependent Theories" submitted for the degree of Doctor of Philosophy (PhD) in the specialty 6D060100 – Mathematics

The thesis work of Zambarnaya Tatyana Sergeevna belongs to the field of the theory of models, one of the important sections of modern mathematical logic. Classification of models of a complete theory up to an isomorphism is one of the main tasks of the Model theory. The spectral questions of complete theories are the important questions in the direction of such a classification. The problem of describing a countable spectrum, that is, the question of classifying countable models of a complete theory up to an isomorphism, is reduced to the Vaught conjecture, which is connected with the Continuum Hypothesis. By efforts of the well-known logicians the Vaught hypothesis have been confirmed for various classes of complete theories: uncountably categorical theories (D. Baldwin - A. Lachlan); omega-stable theories (E. Bouscaren-D. Laskar, S. Shelah); superstable theories (L. Nevelsky, S. Bickler); linear ordered theories expanded by unary predicates (M. Rubin), o-minimal theories (L. Mayer), and quite o-minimal theories (B.Sh. Kulpeshov- S.V. Sudoplatov).

The thesis work of Zambarnaya Tatyana Sergeevna is devoted to the study of countable models of complete countable theories of classes of dependent theories, or equivalently classes of stable theories or classes with the property of strict order without the independence property. The strict order property of a complete theory implies the existence of a formula defining order, not necessarily linear and not necessarily on elements. The PhD candidate was tasked to investigate countable models of complete dependent, unstable theories. The task of the research was divided into two subtasks, which were carried out in two stages: The first task was to find the conditions imposed on the theory so that it had the maximum number of countable non-isomorphic models; The second task was to find a class of complete dependent theories for which the Vaught Hypothesis holds.

The method of research of theories and the method of constructing countable models are based on the Tarski-Vaught theorem and on the theory of (non) - orthogonality of types.

The thesis consists of 8 chapters and an Introduction. The first chapter contains a historical overview, the second chapter contains the necessary basic information, the third and fourth chapters are devoted to general questions of the conditions of maximality of the countable spectrum, which follow from the definition of a finite diagram and the theory of orthogonality of 1-types. The main results of the thesis are contained in Chapters 5-8. The fifth chapter introduces the concept of an extremely trivial 1-type in a theory with definable linear order. It was proved that in this case the theory has the maximum number of countable models.



This theorem strengthens one of the theorems of M. Rubin and is an important step in the direction of solving the Vaught hypothesis.

The sixth chapter contains conditions imposed on a complete theory with a strict order property. The seventh chapter is devoted to linear orders in which it is proved that a linear order with the property of quasi-following has the maximum number of countable models. This is an important theorem for the study of countable models with definable linear order.

The eighth chapter confirms the Vaught hypothesis for a class of weakly o-minimal rank one theories.

As a adviser, I note that in the process of writing her dissertation, Tatyana Sergeevna has grown into a real young research scientist who can successfully work in the theory of models due to her qualities, such as diligence, perseverance, a thorough understanding of model-theoretic properties of complete theories, possession of modern construction techniques calculating models, the ability to creatively solve the problems put before her, finding unexpected solutions.

All the results of the thesis have been tested at many international conferences and seminars, the main results are published by domestic and rated international journals.

I think that the thesis satisfies all the requirements for the theses for the academic degree of Doctor of Philosophy (PhD), and its author deserves to be awarded the degree of Doctor of Philosophy (PhD) in Mathematics.

Scientific supervisor,  
doctor of physical and mathematical sciences,  
corresponding member of the National Academy  
of Sciences of Kazakhstan



B.S. Baizhanov