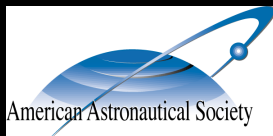


# Third IAA Conference on DYNAMICS AND CONTROL OF SPACE SYSTEMS 2017



**Edited by**  
**Yury N. Razoumny**  
**Filippo Graziani**  
**Anna D. Guerman**  
**Jean-Michel Contant**



**Volume 161**

**ADVANCES IN THE ASTRONAUTICAL SCIENCES**



**DYNAMICS AND CONTROL  
OF SPACE SYSTEMS  
DyCoSS'2017**

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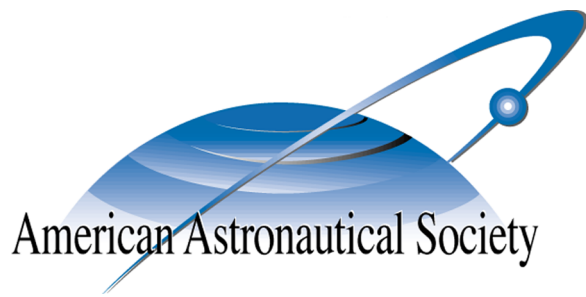
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**Front Cover Illustration:**

The illustration represents the space view of the Earth, International Space Station (ISS) and other satellites in low-earth orbits. Credit: Academy of Engineering, RUDN University.



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## OPTIMAL SYNTHESIS OF SATELLITE ATTITUDE DETERMINATION AND CONTROL SYSTEM'S PARAMETERS

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Kuanysh Alipbayev<sup>†</sup> and Anna Sukhenko<sup>†</sup>

This article discusses the problem of ensuring the quality of transient process of satellite attitude determination and control system. Optimal synthesis control is used for ensuring the required quality of transient process with regard to space systems in world practice. In this paper we consider the approach to optimal synthesis of satellite attitude determination and control system based on the optimal location of the poles of the closed-loop control system. The problem of control of slew from current angular position to desired in minimum time is solved using given approach. Analysis of the numerical solution of this problem has shown the effectiveness of given approach.

### INTRODUCTION

Attitude determination and control system is one of the main subsystems of the satellite which provides its orientation in a predetermined direction during the flight. Mathematical models and algorithms providing the required quality of control processes of satellite rotational motion that are used in attitude determination and control system are one of its important elements. Optimal synthesis control is used for ensuring the required quality of transient process with regard to space systems in world practice. In particular, minimum time, minimum torque and minimum error criteria are used for synthesis of satellite attitude determination and control system's parameters.<sup>1,2</sup>

One of the first serious works devoted to the synthesis of optimal control for satellite is the work of Junkins & Turner where the necessary optimality conditions are formed using the Pontryagin maximum principle.<sup>3</sup> Vadali & Junkins considered the problem of optimal control synthesis for a satellite actuated by reaction wheels. A two-point boundary value problem is constructed on the basis of Pontryagin maximum principle.<sup>4</sup> Bilimoria & Wie made a great contribution to the development of time-optimal control. They considered the problem of time-optimal rotation of satellite with zero initial and final angular velocities.<sup>5</sup>

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At present the field of space technique and associated field of control of space technique are in progress in Kazakhstan. Using of experience of other countries is one of the natural ways of progress.

In this paper we consider the approach to optimal synthesis of satellite attitude determination and control system based on the optimal location of the poles of the closed-loop control system and providing the desired quality of transient processes. The idea of using a certain pole location for the synthesis of control systems is not new, but in comparison with the optimal synthesis it allows to obtain a more versatile engineering approach providing all the indicators of transients quality - time and overshoot.

### MATHEMATICAL MODEL OF SATELLITE ATTITUDE DETERMINATION AND CONTROL SYSTEM

Let us consider the problem of controlling the rotation of a satellite actuated by three reaction wheels from the current angular position to the required in the minimum time.

Dynamics of satellite rotational motion is described with the help of Euler dynamical equations:

$$J\vec{\omega} + \vec{\omega} \times (J\vec{\omega} + J_a \vec{\omega}_a) = \vec{M}, \quad (1)$$

where  $J$  is diagonal (3x3) matrix of satellite inertia tensor;  $\vec{\omega}$  is vector of absolute angular velocity of the satellite in the projections to body coordinate system axes  $C_{xyz}$ ;  $J_a$  is diagonal (3x3) – matrix of reaction wheels inertia tensor;  $\vec{\omega}_a$  is angular velocity vector of reaction wheels set along  $x$ ,  $y$ ,  $z$  axes respectively;  $\vec{M}$  is the vector of reaction wheels control torque in the projections to the axes of body coordinate system  $C_{xyz}$ .

Reaction wheels control torque in (1) is:

$$\vec{M} = -J_a \vec{\omega}_a. \quad (2)$$

Kinematical equations in quaternions are used to describe the satellite rotational motion:

$$\vec{\omega} = 2\vec{Q}^* \otimes \vec{Q}, \quad (3)$$

where  $\vec{Q}$  is quaternion characterizing current angular position of the satellite relative to inertial coordinate system;  $\vec{Q}^*$  is quaternion inverse to  $\vec{Q}$ ;

Reaction wheels dynamics is described by an equation of type:<sup>6</sup>

$$\vec{\dot{\omega}}_a + \frac{1}{T} \vec{\omega}_a = k\vec{U}, \quad (4)$$

where  $\vec{U} = (U_1, U_2, U_3)^T$  is vector of supply voltage of reaction wheels electric motors.

Therefore, satellite attitude determination and control system is described by the system of 10 differential equations of satellite rotational motion (1), (3) and reaction wheels dynamics (4).

## OPTIMAL SYNTHESIS OF THE PARAMETERS OF SATELLITE ATTITUDE DETERMINATION AND CONTROL SYSTEM

Control law of satellite rotational motion is given in the form of linear function:

$$\vec{M} = \vec{M}(\vec{\omega}, \vec{Q}, h, \alpha), \quad (5)$$

where  $h, \alpha$  are unknown parameters required to be determined.

The quality of control system transient processes depends on the values of control law parameters. Let us consider the problem of defining the parameters of control law using the pole placement method.

Pole placement method is used for linear systems described by equation in the form:

$$\dot{\vec{x}} = A\vec{x} + B\vec{u}, \quad (6)$$

where  $\vec{x} \in \mathfrak{R}^{n_x}$  is system state vector,  $\vec{u} \in \mathfrak{R}^{n_u}$  is control vector.

Control law in this case is found in the form:

$$\vec{u} = -K\vec{x}, \quad (7)$$

where  $K = K(h, \alpha)$  is matrix of control law parameters.

Control synthesis problem using pole placement method consists in setting the desired position of characteristic equation roots of the control system providing the required values of dynamical characteristics of closed loop system and finding the matrix  $K$  providing the desired placement of roots.

Taking into account (7), the characteristic equation of closed loop system is written in the form:

$$|sE - A + BK| = s^n + b_{n-1}s^{n-1} + \dots + b_1s + b_0 = 0. \quad (8)$$

Desired location of roots is determined by the polynomial solution:

$$s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0 = 0. \quad (9)$$

Equating the coefficients in (8) and (9) at the same degrees of  $s$  and solving the obtained system of linear equations relative to unknown components of matrix  $K$  one can obtain the control function.

To use the considered theory the equations of satellite rotational motion (1), (3) are linearized in the vicinity of satellite equilibrium position, i.e.  $\vec{Q} = [1, 0, 0, 0]$ ,  $\vec{\omega} = [0, 0, 0]$ . So the satellite angular velocity and angular position are represented as:

$$\vec{Q} = [\hat{q}_0 + \delta q_0, \hat{q}_1 + \delta q_1, \hat{q}_2 + \delta q_2, \hat{q}_3 + \delta q_3], \quad (10)$$

$$\vec{\omega} = [\hat{\omega}_1 + \delta \omega_1, \hat{\omega}_2 + \delta \omega_2, \hat{\omega}_3 + \delta \omega_3], \quad (11)$$

where  $\delta q_0, \delta q_1, \delta q_2, \delta q_3, \delta \omega_1, \delta \omega_2, \delta \omega_3$  are variables characterizing deviations of satellite angular position and angular velocity from the equilibrium position.

Substituting expressions (10), (11) into satellite rotational motion equations (1), (3) and rejecting the terms higher than the first order we obtain:

$$\begin{aligned}\dot{\omega}_1 &= \frac{1}{J_1} [(J_2 - J_3)(\hat{\omega}_2 \delta \omega_3 + \delta \omega_2 \hat{\omega}_3) - J_{a_3} \omega_{a_3} \delta \omega_2 + J_{a_2} \omega_{a_2} \delta \omega_3 + \delta M_1], \\ \dot{\omega}_2 &= \frac{1}{J_2} [(J_3 - J_1)(\hat{\omega}_1 \delta \omega_3 + \delta \omega_1 \hat{\omega}_3) - J_{a_1} \omega_{a_1} \delta \omega_3 + J_{a_3} \omega_{a_3} \delta \omega_1 + \delta M_2], \\ \dot{\omega}_3 &= \frac{1}{J_3} [(J_1 - J_2)(\hat{\omega}_1 \delta \omega_2 + \delta \omega_1 \hat{\omega}_2) - J_{a_2} \omega_{a_2} \delta \omega_1 + J_{a_1} \omega_{a_1} \delta \omega_2 + \delta M_3]\end{aligned}\quad (12)$$

$$\begin{aligned}\delta \ddot{q}_0 &= \frac{1}{2} (-\hat{\omega}_1 \delta q_1 - \delta \omega_1 \hat{q}_1 - \hat{\omega}_2 \delta q_2 - \delta \omega_2 \hat{q}_2 - \hat{\omega}_3 \delta q_3 - \delta \omega_3 \hat{q}_3), \\ \delta \ddot{q}_1 &= \frac{1}{2} (\hat{\omega}_1 \delta q_0 + \delta \omega_1 \hat{q}_0 + \hat{\omega}_3 \delta q_2 + \delta \omega_3 \hat{q}_2 - \hat{\omega}_2 \delta q_3 - \delta \omega_2 \hat{q}_3), \\ \delta \ddot{q}_2 &= \frac{1}{2} (\hat{\omega}_2 \delta q_0 + \delta \omega_2 \hat{q}_0 + \hat{\omega}_1 \delta q_3 + \delta \omega_1 \hat{q}_3 - \hat{\omega}_3 \delta q_1 - \delta \omega_3 \hat{q}_1), \\ \delta \ddot{q}_3 &= \frac{1}{2} (\hat{\omega}_3 \delta q_0 + \delta \omega_3 \hat{q}_0 + \hat{\omega}_2 \delta q_1 + \delta \omega_2 \hat{q}_1 - \hat{\omega}_1 \delta q_2 - \delta \omega_1 \hat{q}_2).\end{aligned}\quad (13)$$

Further taking into account that  $\vec{Q} = [1, 0, 0, 0]$ ,  $\vec{\omega} = [0, 0, 0]$ , from (12), (13) we obtain:

$$\begin{aligned}\dot{\omega}_1 &= \frac{1}{J_1} [-J_{a_3} \omega_{a_3} \delta \omega_2 + J_{a_2} \omega_{a_2} \delta \omega_3 + \delta M_1], \\ \dot{\omega}_2 &= \frac{1}{J_2} [-J_{a_1} \omega_{a_1} \delta \omega_3 + J_{a_3} \omega_{a_3} \delta \omega_1 + \delta M_2], \\ \dot{\omega}_3 &= \frac{1}{J_3} [-J_{a_2} \omega_{a_2} \delta \omega_1 + J_{a_1} \omega_{a_1} \delta \omega_2 + \delta M_3]\end{aligned}\quad (14)$$

$$\begin{aligned}\delta \ddot{q}_0 &= 0, \\ \delta \ddot{q}_1 &= \frac{1}{2} \delta \omega_1, \\ \delta \ddot{q}_2 &= \frac{1}{2} \delta \omega_2, \\ \delta \ddot{q}_3 &= \frac{1}{2} \delta \omega_3.\end{aligned}\quad (15)$$

As a result reducing the system of equations (14), (15) to the form (6), we obtain:

$$\begin{bmatrix} \delta\dot{q}_1 \\ \delta\dot{q}_2 \\ \delta\dot{q}_3 \\ \delta\dot{\omega}_1 \\ \delta\dot{\omega}_2 \\ \delta\dot{\omega}_3 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 0 & -\frac{J_{a3}\omega_{a3}}{J_1} & \frac{J_{a2}\omega_{a2}}{J_1} \\ 0 & 0 & 0 & \frac{J_{a3}\omega_{a3}}{J_2} & 0 & -\frac{J_{a1}\omega_{a1}}{J_2} \\ 0 & 0 & 0 & -\frac{J_{a2}\omega_{a2}}{J_3} & \frac{J_{a1}\omega_{a1}}{J_3} & 0 \end{bmatrix} \begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ \delta \omega_1 \\ \delta \omega_2 \\ \delta \omega_3 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{J_x} & 0 & 0 \\ 0 & \frac{1}{J_y} & 0 \\ 0 & 0 & \frac{1}{J_z} \end{bmatrix} \begin{bmatrix} \delta M_1 \\ \delta M_2 \\ \delta M_3 \end{bmatrix}, \quad (16)$$

where

$$\begin{bmatrix} \delta M_1 \\ \delta M_2 \\ \delta M_3 \end{bmatrix} = -K \begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ \delta \omega_1 \\ \delta \omega_2 \\ \delta \omega_3 \end{bmatrix} = - \begin{bmatrix} \alpha_1 & 0 & 0 & h_1 & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & h_2 & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 & h_3 \end{bmatrix} \begin{bmatrix} \delta q_1 \\ \delta q_2 \\ \delta q_3 \\ \delta \omega_1 \\ \delta \omega_2 \\ \delta \omega_3 \end{bmatrix}. \quad (17)$$

Let us set the desired location of characteristics equation roots using some distributions wide spread in practice: binomial distribution and Butterworth distribution.

The roots of the Newton polynomial are taken as the desired roots of the characteristic equation for the binomial distribution. For example for the system of the 6th order there are taken the roots of polynomial:

$$s^6 + 6\rho s^5 + 15\rho^2 s^4 + 20\rho^3 s^3 + 15\rho^4 s^2 + 6\rho^5 s + \rho^6 = 0, \quad (18)$$

where  $\rho = \frac{t_n}{t_r}$  is dimensionless scale of conversion from the normalized transient time  $t_n$  to the real time of transient process  $t_r$ .

That is, all the roots of the characteristic equation are chosen to be real and having the same values  $s = -\rho < 0$ .

The Butterworth distribution is determined by the location of the roots of the n-th order characteristic equation in the left half-plane on the circle of radius  $\rho$  at the vertices of a regular 2n-gon inscribed in it, symmetric with respect to the real and imaginary axes. For example for the system of the 6th order there are taken the roots of polynomial:<sup>7</sup>

$$s^6 + 3.86\rho s^5 + 7.46\rho^2 s^4 + 9.13\rho^3 s^3 + 7.46\rho^4 s^2 + 3.86\rho^5 s + \rho^6 = 0. \quad (19)$$

Let us set the satellite moments of inertia  $J = [0.04088; 0.04088; 0.01116]$  kgm<sup>2</sup>, reaction wheels moment of inertia  $J_M = [0.00000011; 0.00000011; 0.00000011]$  kgm<sup>2</sup>, reaction wheels motors armature resistance  $R = 38 \Omega_M$ , EMF coefficient of reaction wheels motors  $k_e = 0.00708 Bc$ .

In order to develop an engineering methodology for synthesizing the parameters of control law according to the specified characteristics of transient processes it is convenient to use the normalized characteristic equation of a closed loop control system. This allows us to determine the normalized time and the overshoot of the transient process for a given distribution of the roots of the characteristic equation from the results of numerical simulation.

To do this let's specify  $\rho = 1$  corresponding to the case of normalized polynomials (18), (19). As a result of the numerical solution the following values of components of the matrix  $K$  were obtained.

a) in case of binomial distribution:

$$\begin{aligned} \alpha_1 = 0.0817, \alpha_2 = 0.0817, \alpha_3 = 0.0223, \\ h_1 = 0.0814, h_2 = 0.0821, h_3 = 0.0223. \end{aligned} \quad (20)$$

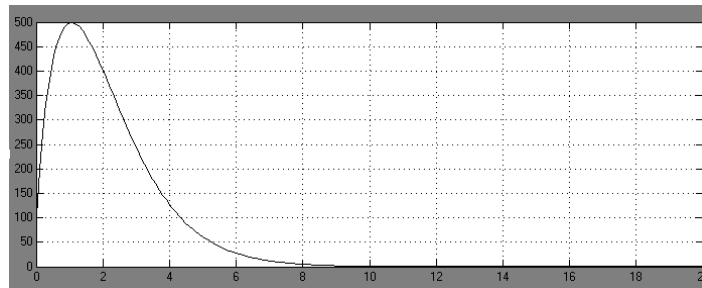
b) in case of Butterworth distribution:

$$\begin{aligned} \alpha_1 = 0.0817, \alpha_2 = 0.0817, \alpha_3 = 0.0223, \\ h_1 = 0.0583, h_2 = 0.0784, h_3 = 0.0057. \end{aligned} \quad (21)$$

Numerical simulation has been performed for the analysis of the efficiency of the control parameters obtained above. Initial angular position of satellites is  $\varphi = 40^\circ, \theta = -50^\circ, \psi = 10^\circ$  ( $\vec{Q} = [0.86100, 0.27418, -0.42263, -0.06976]$ ),  $\omega_1 = 0, \omega_2 = 0, \omega_3 = 0$ . Required angular position of satellite is  $\varphi = 0^\circ, \theta = 0^\circ, \psi = 0^\circ$  ( $\vec{Q} = [1, 0, 0, 0]$ ).

**Table 1. Magnitude of the vector part of quaternion.**

Time, sec	Magnitude of the vector part of quaternion in case of binomial distribution	Magnitude of the vector part of quaternion in case of Butterworth distribution
0	0,508592	0,508592
1	0,402255	0,39496
2	0,245954	0,221578
3	0,130365	0,106099
4	0,063208	0,054649
5	0,028967	0,025671
7	0,012791	0,01065
8	0,005505	0,011902
9	0,002326	0,007319
10	0,000969	0,000757



**Figure 1. Supply voltage of reaction wheels motors when  $\rho = 1$ .**

The results of numerical simulation when using parameters (20), (21) are given in Table 1. As can be seen from table the value of magnitude of the quaternion vector part reaches the border of 3% tube of deviation from the final angular position within 5 seconds. However, the value of the supply voltage of the reaction wheel motor is much higher than its nominal value of 18 V as it is shown at the Figure 1. Consequently, the real satellite control system can not provide such a short transient time due to the limited supply voltage and power of the reaction wheel motors.

In order to take into account the limited power of the reaction wheel motors a number of numerical experiments were carried out taking as the initial approximation  $\rho$  equal to the nominal value of the supply voltage (18V) divided by the peak value of the supply voltage (500 V).

As result of numerical solution we obtained the following values of components of matrix  $K$  :

a) for the binomial distribution:

$$\begin{aligned} \alpha_1 &= 0.0000718, \alpha_2 = 0.0000752, \alpha_3 = 0.0000201, \\ h_1 &= 0.0025902, h_2 = 0.0026800, h_3 = 0.0005700. \end{aligned} \quad (22)$$

b) for the Butterworth distribution:

$$\begin{aligned} \alpha_1 &= 0.0000735, \alpha_2 = 0.0000735, \alpha_3 = 0.0000201, \\ h_1 &= 0.0005855, h_2 = 0.0024262, h_3 = 0.0004701. \end{aligned} \quad (23)$$

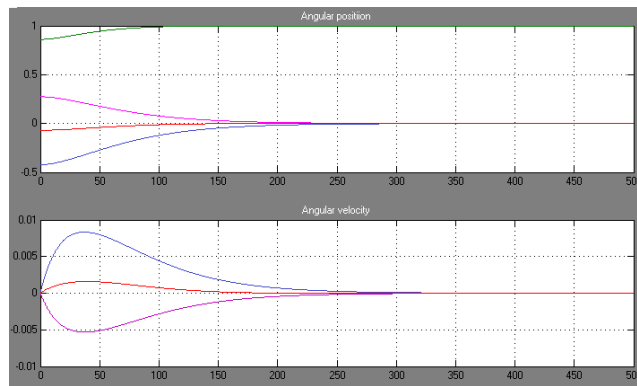


Figure 2. Angular position and angular velocity of satellite in case of binomial distribution.

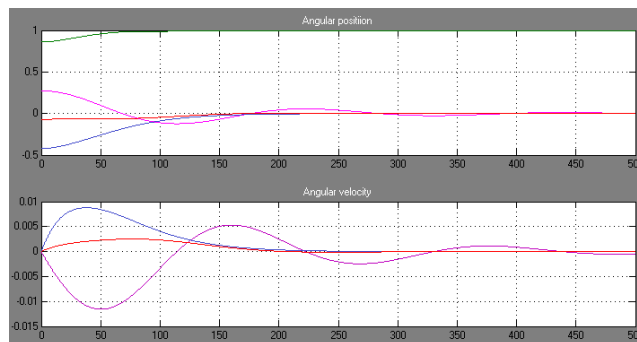


Figure 3. Angular position and angular velocity of satellite in case of Butterworth distribution.

**Table 2. Magnitude of the vector part of quaternion.**

Time, sec	Magnitude of the vector part of quaternion in case of binomial distribution	Magnitude of the vector part of quaternion in case of Butterworth distribution
167	0,040959	0,031905
168	0,040153	0,029425
169	0,039361	0,026998
170	0,038585	0,024636
171	0,037824	0,022355
172	0,037078	0,020174
173	0,036346	0,018123
174	0,035628	0,016238
175	0,034924	0,014572
176	0,034233	0,013193
177	0,033556	0,012178
178	0,032892	0,011605
179	0,03224	0,011518
180	0,031602	0,011905
181	0,030975	0,012702
182	0,030361	0,013819
183	0,029759	0,015165
184	0,029168	0,016669
185	0,028589	0,018275
186	0,028022	0,019943
187	0,027465	0,021645
188	0,026919	0,02336
189	0,026384	0,025074
190	0,025859	0,026775
191	0,025345	0,028454
192	0,024841	0,030105
193	0,024346	0,031723
194	0,023861	0,033304
195	0,023386	0,034845

Results of numerical simulation when using parameters (22), (23) are given at the Figures 2,3 and Table 2. Table 2 shows that in case of Butterworth distribution magnitude of quaternion vector part reaches the border of 3% tube of deviation from the final angular position much faster than in case of binomial distribution, but near to the end of time interval [150,200] it leaves the



3% tube. Consequently the binomial distribution is most preferable for determining the parameters of the control law for the rotational motion of the considered satellite.

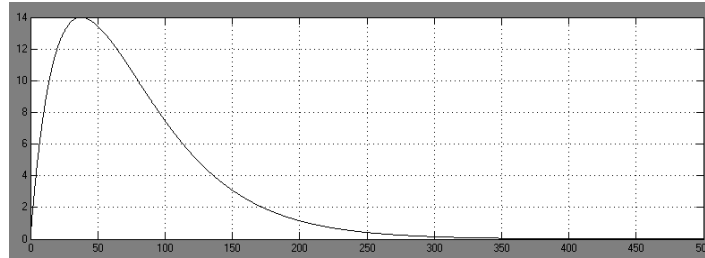


Figure 4. Supply voltage of reaction wheel motor when  $\rho = 0.03$ .

Figure 4 shows that supply voltage of the reaction wheel motors is within the limits of permissible value (less than 18V) when using control parameters (22), (23).

## CONCLUSION

In this article the problem of ensuring the quality of transient process of satellite attitude determination and control system is considered. It is solved using pole placement method for defining the unknown parameters of control law of satellite rotational motion. Wherein the wide spread distributions have been used for the setting of location of characteristic equation roots of closed loop control system in the complex half-plane. According to the obtained results of numerical modeling of satellite controlled rotational motion with the calculated parameters it has been determined that given approach can be effectively used for the synthesis of control system using its the required dynamical characteristics.

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